АВТОМАТИКА
и
ТЕЛЕМЕХАНИКА



*Volume 19, No. 9*

*September 1958*

SOVIET INSTRUMENTATION AND
CONTROL TRANSLATION SERIES

# Automation and Remote Control

(The Soviet Journal *Avtomatika i Telemekhanika* in English Translation)

■ This translation of a Soviet journal on automatic control is published as a service to American science and industry. It is sponsored by the Instrument Society of America under a grant in aid from the National Science Foundation, continuing a program initiated by the Massachusetts Institute of Technology.

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Publication of *Avtomatika i Telemekhanika* in English translation started under the present auspices in April 1958 with Russian Vol. 18, No. 1 of January 1957. Translation of Vol. 18 has now been completed. The twelve issues of Vol. 19 will be published in English translation by mid-1959.

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Instrument Society of America
313 Sixth Avenue, Pittsburgh 22, Penna.

*Translated and printed by Consultants Bureau, Inc.*

*Volume XIX No. 9 – September 1958*

*English Translation Published June 1959*

# Automation and Remote Control

## *The Soviet Journal Avtomatika i Telemekhanika in English Translation*

***Reported circulation of the Russian original 8,000.***

***Avtomatika i Telemekhanika is a Publication of the Academy of Sciences of the USSR***



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AUTOMATION AND REMOTE CONTROL

Volume 19, Number 9 September 1958

# CONTENTS

# CONTROL BASED ON THE PRINCIPLE OF A SELF-ADJUSTING PROGRAM

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(Moscow)

An example of the control of objects subjected to repeated disturbances is given. Compensation for these disturbances by means of special response at the program input is suggested. Stability conditions are determined. The problem of stabilizing hot-rolled steel sheets is analyzed.

Normal closed-loop automatic control systems which contain pure time-delay elements respond in a certain sense satisfactorily only to those components of disturbing or control actions whose duration exceeds the time delay $\tau$.

The present paper deals with the principle of automatic control-system design which ensures the required response to the input signal components whose duration can be both greater or smaller than $\tau$. Such systems are known as self-adjusting program systems.

These systems are designed to work only with certain types of disturbing or control actions, such as those encountered, for instance, in mass production processes, i.e., in processes which are definitely repetitive, cyclic. Such effects (whose precise definition will be given below) are called cyclic.

In order to make further discussion more concrete we shall examine the problem of stabilizing the thickness of a steel sheet hot-rolled on a thin-sheet continuous rolling mill. The solution of this problem requires the application of the self-adjusting program method of control.

## The Problem of Stabilizing the Thickness of a Hot-Rolled Sheet

It is known that the thickness of a hot-rolled steel sheet does not remain constant along its length. This fact is due to the uneven pressure exerted by the rolled metal on the mill stand rolls, causing elastic strains in the housings and rolls, which lead to an alteration in the spacing of rolls. It is possible to eliminate the longitudinal differences in thickness by controlling the stand pressure mechanism in such a way as to compensate for the strains in the housings and rolls by displacing the rolls.

Thus, the solution of the problem of decreasing the unevenness in the thickness of the sheets is reduced to finding a method for controlling the stand pressure mechanism.

The most obvious solution of this problem would appear, at first sight, to be the control of the pressure mechanisms as functions of the measured deviation of the rolled metal thickness, i.e., control in a closed system: sheet, thickness gauge, pressure mechanism, sheet. The hot-rolling mill is characterized, however, by a very large time interval (and hence a large distance along the strip length) between the moment (place) of application of the correcting pressure and the moment (place) of the thickness measurement. This fact prevents satisfactory results being obtained by means of a normal closed-loop automatic control system.

Let us examine the possibility of controlling the pressure mechanisms by some other means. Let us denote the deviation of the thickness from its nominal value by $\Delta h$. For each rolled strip the change in $\Delta h$ along the strip length can be represented in the form of the relation

$$\Delta h = f(l),$$

where $l$ is the current value of the strip length, counted from its beginning, "head." The form of function $f(l)$ is determined by the sum of all the physical and technological factors which determine the rolling process. These factors possess varying degrees of stability. Some change during the rolling of one strip, thus changing its thickness (temperature drop between the "head" and the "tail" of the strip, absence of tension at the "tail" etc.); others change from strip to strip (the mean temperature of the slab, roller surface condition, etc.); yet others remain constant for all the strips of the same consignment. By consignment we understand a group of strips rolled one after the other from the same grade of steel to the same nominal geometrical dimensions. In practice, of course, the rolling is always done in consignments.

The study of the difference in thickness characteristics [1 and 2] has shown that the aggregate of functions $f(l)$ taken for strips of the same consignment possess many similar characteristics.

Function $f(l)$ can be represented in the form

$$f(l) = f_c(l) + f_r(l)$$

where $f_c(l)$ is a function common to all the strips of the same consignment, and which determines the "regular" variations of the strip thickness along its length characteristic to all or the majority of strips of that consignment, $f_r(l)$ is the random function which determines the deviation of $f(l)$ of any strip from $f_c(l)$.

The study of function $f(l)$ [1 and 2] shows that the component "weight" of function $f_c(l)$ in function $f(l)$ is large.

The elimination of component $f_c(l)$ would decrease the strip thickness irregularities to between $1/3$ and $1/4$ of their initial value [2]. If function $f_c(l)$ were known to us, the irregularities determined by it could, obviously, be eliminated by controlling the stand pressure mechanisms according to a predetermined program calculated for the given $f_c(l)$. An attempt was made to introduce program control of the pressure screws [1]. According to this arrangement when a slab of a certain grade of steel and certain dimensions had to be rolled into strips of given dimensions, the pressure mechanisms were subjected during rolling to the control of a corresponding predetermined regulation program.

It is pointed out in [1], however, that this attempt was not effective owing to the variation of function $f_c(l)$ for different consignments of the same grade of rolled steel. Moreover, in certain cases variations of $f_c(l)$ within the same consignment were also found possible.

In order to obtain satisfactory results it is obviously necessary that any changes in $f_c(l)$ should be followed by corresponding changes in the program control of the pressure mechanisms. An automatic production of the required program we shall call self-adjustment of the program.

Let us now point out an important fact for subsequent reasoning, that in investigating the movement of rolled strips with respect to any fixed point of the rolling mill the function of the current length $f(l)$ can be represented as a function of time $\underline{t}$. In fact, coordinate $l$ of any cross section of the strip can be related to the time at which that cross section passes a given point of the mill. The speed of rolling of any given consignment is practically constant, and variables $l$ and $\underline{t}$ bear a linear relationship to each other. It should also be noted that function $f(t)$, obtained by substituting variable $l$ by $\underline{t}$ in (1), describes the process of elastic strain changes in stands at the finishing stage.

## Cyclic Action

Elastic strain changes in stands are an example of a cyclic disturbing action. Under this term we understand a disturbing action described by the function of time $f(t)$ which satisfies the following conditions:

1. $f(t)$ consists of consecutive cycles each of which can be divided into an effective movement and a pause.

2. The behavior of $f(t)$ in the effective movement portions of two arbitrarily selected adjacent cycles, the n-th and the (n + 1)-th are subject to condition

$$f_n(t_n - t_{0n}) \approx f_{n+1}(t_{n+1} - t_{0n+1}) \tag{2}$$

with $t_n - t_{0n} = t_{n+1} - t_{0n+1} = t^*$, where $t_{0n}$ and $t_{0n+1}$ denote the instants at which the effective movements of the n-th and the (n + 1)-th cycle begin respectively, and $t_n$ and $t_{n+1}$ denote the current values of time during the n-th and the (n + 1)-th effective movements.

The approximate equality in (2) is understood to mean that the regulated object output coordinate deviation from its required value under the influence of disturbance $\Delta(t^*) = f_{n+1}(t^*) - f_n(t^*)$ does not exceed the desired limits of regulation accuracy.

The quasi-periodic function $f(t) \approx f(t + T)$, where T is a constant (period), is a particular case of a cyclic function.

3. Within the limits of effective motion, $f_n(t^*)$ represents a sufficiently smooth function in the sense that it can be approximated, with an accuracy acceptable for practical purposes (see preceding paragraph), to a step function $\psi_n(t^*)$. The value of $\psi_n(t^*)$ in the intervals of time between $t^* = (k-1)T$ and $t^* = kT$ has a value of $\psi_{nk}$, where T is the duration of a step (a predetermined value), k = 1, 2 . . . and quantity $\psi_{nk}$ is determined by relationship

$$\psi_{nk} = \frac{1}{T} \int\limits_{T(k-1)}^{T_k} f_n(t^*)\, dt^* .$$

4. There is the possibility of receiving, without any time delay, information on the beginning and end of each effective movement.

5. The range of variations of the effective movement duration is limited and known in advance.

The behavior of the function during pauses and the duration of pauses are not subject to any limitations.

## Principles of Operation of a Self-Adjusting Program System

Let cyclic disturbance $f(t)$, applied to the regulated object during an effective motion period, lead to a deviation of the output coordinate (to error) $\delta(t)$. Obviously, $\delta(t)$ will also be a cyclic function.

Let there be in the regulated object (or the error measuring circuit) a delay $\tau$. In this case, as it was stated above, the use of a regulator with a self-adjusting program will lead to a decrease not only in the low frequency component of $\delta(t)$, but also in components of a duration smaller than $\tau$, a result which cannot be obtained by the usual control methods.

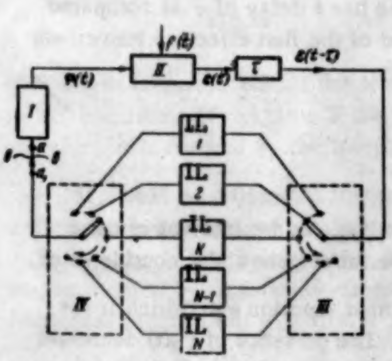


Fig. 1

The principle of the self-adjusting program control consists in bringing into the system during each n-th effective movement an additional action (program) $\psi_n(t^*)$ which is selected in such a way as to compensate for the disturbing action at the output of the regulated object.

The calculation of the program is based on the data of measured errors, obtained during the operation of the system over cycles preceding the one in question. The calculation of the program amounts to determining the expected disturbing action of $f_n(t^*)$ on its n-th effective movement, on the basis of the known disturbing actions $f_{n-1}(t^*)$, $f_{n-2}(t^*)$ etc. In the case when $f(t)$ consists of a regular component $f_c(t)$ and a random one $f_r(t)$, the calculation of the program is reduced to extracting from the functions $f_{n-1}(t^*)$, $f_{n-2}(t^*)$, etc. the regular function $f_c(t)$.

The principle of self-adjusting program control can be achieved by means of connecting into the feedback circuit a computer, which acts as a modified synchronous filter [3 and 4]. Above device (Fig. 1) consists of N integrating links (IL) an input switch III and an output switch IV. The output switch connects in turn all the IL outputs to the regulator input.

The regulator is set to ensure, with a normal closed control system, consisting of regulator I and regulated object II with a delay $\tau = 0$, the required reaction to a given disturbing action $f(t)$.

At the instant of the beginning of an effective movement the regulator input is connected to the output of IL 1. After a predetermined interval of time T, which we shall call integration interval, the regulator input is instantaneously switched over to the IL 2 output. The IL 1 output circuit is simultaneously disconnected. After a time interval of 2T from the beginning of the effective movement the output of the third IL is connected and that of the second IL disconnected, etc., until the signal of the end of an effective movement disconnects the regulator input during a pause, followed by the repetition of the process described above when the signal of the beginning of the next effective movement is received.

The input switch, which connects in turn all the IL inputs to the error measuring circuit, follows the same sequence of operations as the output switch, but with a time lag of $\tau$.

The duration of the integration intervals T is chosen to conform to the inequality

$$T < \tau. \tag{3}$$

This condition can, obviously, always be fulfilled in the design of the system.

Let us henceforth assume that the following relationship exists between the minimum duration of the cycle $T_{min}$ and $\tau$

$$\tau < T_{\min} - T, \tag{4}$$

which is not an indispensable condition for obtaining a workable system, but which greatly facilitates its investigation.

In fulfilling (3) and (4) the input and output of each IL are never connected simultaneously. Therefore, the signal taken from each IL is a constant quantity, determined by the charge accumulated in IL at the instant its input was last disconnected. The sequence of control signals arriving at the regulator input during one effective movement represents step function $\psi(t)$ which serves as a program for the regulator.

For simplifying the description of the operation of the device let us assume that the transfer function of the series-connected regulator and object is equal to unity*, and the time constant of IL is equal to T/K, where K = 1.

Let the charges of all the IL be equal to zero at the beginning of the first effective movement. In this case, during the first effective movement, error $\epsilon_1(t) = \delta_1(t) - \psi_1(t) = \delta_1(t) - 0 = \delta_1(t)$ will appear at the output of the object. Owing to the delay in the measuring circuit this error will arrive at the output switch in the form of the signal $\epsilon_1(t-\tau)$. Owing to the fact that the input switch also has a delay of $\tau$ as compared with the effective movement cycle, the IL with index $\underline{i}$ will have at the end of the first effective movement a charge of

$$Q_{1i} = \frac{K}{T} \int_{(i-1)T}^{iT} \varepsilon_1(t)\, dt = K\varepsilon_{1i\,m},$$

where $\epsilon_{1im}$ is the mean error at the i-th interval of the effective movement, i.e., at the interval of time between instants $(i-1)T$ and $iT$ reckoned from the beginning of the effective movement under consideration.

During the second effective movement there appears at the regulator input reaction $\psi_2(t)$ which is represented by a step function with an amplitude of the i-th step equal to $Q_{1i}$. The presence of $\psi_2(t)$ decreases the error $\epsilon_2(t)$ which appears at the object output during the second effective movement. Thus, the error at the i-th interval of the second effective movement will be $\epsilon_{2i}(t) = \delta_{2i}(t) - K\epsilon_{1im}$. At the end of the second effective movement the i-th interval will receive a charge of $Q_{2i} = Q_{1i} + K\epsilon_{2im}$. Hence, the amplitude of the i-th step reaction $\psi_3(t)$ will be equal to $Q_{2i}$ and the error at the third effective movement will be decreased to $\epsilon_{3i}(t) = \delta_{3i}(t) - Q_{2i} = \delta_{3i}(t) - K(\epsilon_{1im} + \epsilon_{2im})$ etc.

---

* In this case it is equivalent to the condition that the transients are damped out in the regulator in a time interval slightly smaller than T.

After a number of cycles n depending on the value of coefficient K the transitional process of program compilation comes to an end in the sense that the error at the output of the object is decreased to a value determined by the accuracy of approximation of the step function $\psi_n(t)$ and the continuous function $\delta_n(t)$ as well as by the amount of variation in $\delta(t)$ from one cycle to another. Changes in function $\delta(t)$ cause continuous program readjustments, carried out in the way described above.

Let us return to the problem of stabilizing the thickness of the hot-rolled sheet.* In this instance the beginning and end of the effective movement correspond to the instants at which the rolled metal enters and leaves the controlled stand. The division of the effective movement into integration intervals corresponds to an arbitrary division of the strip length into equal intervals. The signal received from the i-th IL is an instruction to the next pressure mechanism drive for setting the required spacing between the rolls for the passage of the i-th interval of the strip. The thickness deviations measured by a noncontact micrometer over the i-th interval are impressed onto the input of the i-th IL.

Owing to the averaging of the deviations measured along the length of the interval and to the accumulation in the integrators of data on the mean deviations of all the previously rolled strips of the same consignment, a program is being gradually built up in the computer for correcting pressures, which would eliminate the thickness irregularity component represented by the function $f_c(l)$. At the same time the random deviations represented by function $f_r(l)$ are filtered out and have but a small effect on the accuracy of the program. The degree to which the random function $f_r(l)$ influences the program is determined by the choiceof the system amplification coefficient K.**

## Stability of a Self-Adjusting Program System

The stability of a self-adjusting program system will be considered in the normally accepted sense.

Investigations of the stability of a self-adjusting program system are impeded by the fact that in the general form the system has variable parameters. In order to overcome these difficulties let us assume that the system under investigation is subjected to disturbances with a constant duration of the cycle $T_c$, that there are no pauses in the disturbance and the cycle is divided into N equal integration intervals, i.e., that the following condition is being fulfilled

$$T_c = T_{\text{eff. move.}}, \qquad T_c = NT. \tag{5}$$

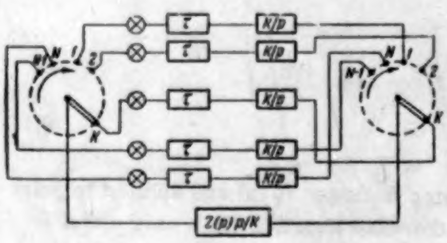


Fig. 2

In this case the investigation of the stability of the system is equivalent to that of the stability of a multilink system formed by N similar circuits, each of which consists of an element with delay $\tau$, an integrating link and a lagged regulator which in turn serves (connects to) all the N circuits (Fig. 2). The switching of any of the circuits is repeated at intervals of NT consisting of the time T the circuit is connected and the time $(N-1)T$ it is disconnected. A link between the circuits is ensured by the inertia of the common regulator.

It should be noted that the criterion of stability found for condition (5) can in many cases be used for determining the required conditions for stability under disturbances of a more general type, i.e., when $T_c > T_{\text{eff. move.}}$. Let the regulator, the regulated object and the IL of the system under investigation (Fig. 1) be represented by transfer functions $Z_1(p)$, $Z_2(p)$, and $K/p$, respectively, where $Z_1(p)$ and $Z_2(p)$ are rational fractional functions of p and K is a real number.

Let us call operator

* For the sake of simplicity,regulation by means of a single stand of the finishing stage is examined. A system of control by means of several stands of the finishing stage is described in [2].

** The determination of the degree to which random deviations influence the speed of compiling and the accuracy of the program and the evaluation in this connection of an optimum value for K is a subject for a separate investigation.

$$Z(p)=\frac{P(p)}{Q(p)}=Z_1(p)\,Z_2(p)\,\frac{K}{p}\,. \tag{6}$$

a transfer function of the linear part of the system.

Let us consider that all the poles of Z(p) are located on the left half-plane and the imaginary axis and for simplifying further calculations let us assume that all these poles are simple ones.

It was previously shown that in fulfilling conditions (3) and (4) the signal, arriving at the regulator input, represents a sequence of rectangular pulses each of which has a duration of T. Such a signal form permits one to use method (5).

Let us now divide the system under investigation (Fig. 1) along line bb and let us take point a as the input of an open system and point $a_1$ as its output. Let us take as a reference point for time t the instant the IL 1 output is closed.

Let a unity amplitude pulse be impressed on the input of the open system at time t = 0 and let the pulse have a duration equal to the integration interval T.

The reaction at the output due to this pulse will be represented by a step function H(n), constant for all the values of dimensionless time $\bar{t}$ ($\bar{t}=t/T$) within the limits of $n-1 \le \bar{t} \le n$. By applying a discrete Laplace transformation to H(n) we obtain its image $H^*(q) = D\{H(n)\}$.

Let us note the important circumstance that owing to the identity of parameters and conditions of operation of all the integrating links, the reaction of the open system to a single signal which starts at instant m and ends at instant m + 1, where m is a whole number, is represented by function H(n − m). Hence, it follows that the multiplication theorem is applicable to the image of H(n). The reaction at the output of the system due to the application of step function $U_{in}(n)$ to its input will be expressed in the sphere of images by the equation

$$U^*_{out}(q) = H^*(q)\,U^*_{in}(q)$$

By applying (5) we obtain an equation for a closed system

$$U^*_{out}(q)=\frac{H^*(q)}{1+H^*(q)}\,U^*_0(q), \tag{7}$$

where $U^*_0(q)$ is the image of the external reaction represented by the step function $H_0(n)$ and applied to point a of the closed system. Point a is not a real input to the system and therefore Equation (7) is only useful for investigating stability.

A closed system, as is known, will be stable only if all the poles of expression $H^*(q)/[1+H^*(q)]$ lie in the left half-plane of the complex plane q. From above a criterion of stability can be derived [5].

Let the open system transfer function $H^*(q)$ have no poles in the right half-plane. The condition of stability for a closed system is that the locus of the frequency characteristic of $H^*(jx)$ with x varying between $-\pi$ and $\pi$ does not enclose point (− 1, 0 j).

An open system transfer function is

$$H^*(q)=\left[c_1'+\sum_{\nu=2}^{l} c_\nu\,\frac{(e^{q_\nu}-1)(e^{q}-1)}{e^{q}-e^{q_\nu}}\right]\frac{1}{e^{qN}-1}\,. \tag{8}$$

Expressions which show the relation between the coefficients and parametersof the system and the derivation of Equation (8) are given in the appendix.

It can easily be seen from (8) that in the region of the basic poles [from $\mathrm{Im}(q) = -\pi$ to $\mathrm{Im}(q) = \pi$], $H^*(q)$ has $(l-1)$ simple poles at points $q = q_\nu$ and N poles on the imaginary axis at $q = 2\pi jn/N$ where $n = 0, 1, 2, \ldots, N-1$. Hence, if Z(q) has no poles in the right half-plane, $H^*(q)$ has no poles there either.

On the basis of (8) it is possible to construct a locus of the frequency characteristic of $H^*(jx)$:

$$H^*(jx) \approx \left[ c_1' + \sum_{\nu=2}^{l} c_\nu \frac{(e^{q_\nu}-1)(e^{jx}-1)}{e^{jx}-e^{q_\nu}} \right] \frac{1}{e^{jxN}-1}. \tag{9}$$

Example: Let the transfer function of the linear part be

$$Z(p) = \frac{K}{p(T_1 p + 1)}.$$

Hence,

$$c_1' = KT, \quad c_\nu = c_2 = KT_1, \quad q_\nu = q_2 = -T/T_1 = -\sigma.$$

Assuming that $T > T_1$, let us use the approximate formula (9):

$$H^*(jx) = T_1 K \left[ \frac{T}{T_1} + (e^{-\frac{T}{T_1}} - 1) \frac{e^{jx}-1}{e^{jx} - e^{-\frac{T}{T_1}}} \right] \frac{1}{e^{jxN}-1} \tag{10}$$

The locus (Fig. 3)

$$(e^{-\frac{T}{T_1}} - 1) \frac{e^{jx}-1}{e^{jx} - e^{-\frac{T}{T_1}}}$$

represents a semi-circle in the third quarter with a diameter consisting of a part of the negative real axis, which it crosses at points 0 (when $x = 0$) and at $2\operatorname{th}(T/2T_1)$ (when $x = \pi$).

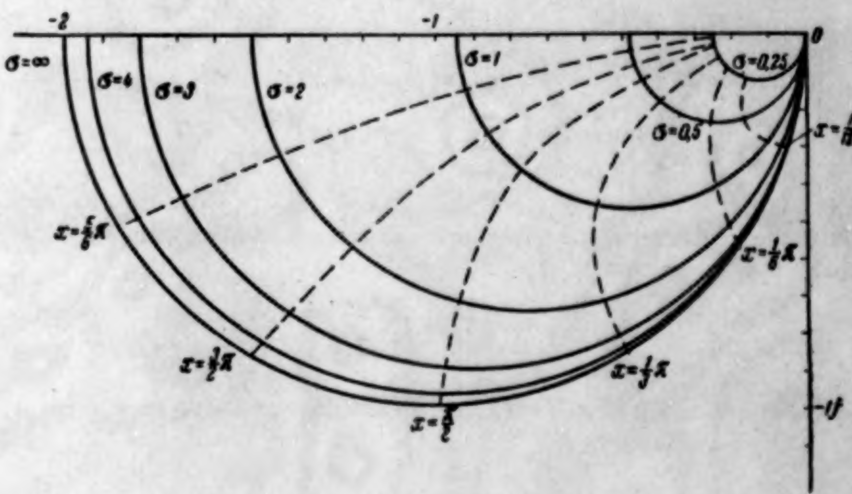


Fig. 3

The vector locus

$$\overline{A}(x) = A(x)\, e^{j\alpha(x)} = \frac{T}{T_1} + (e^{-\frac{T}{T_1}} - 1) \frac{e^{jx}-1}{e^{jx} - e^{-\frac{T}{T_1}}}$$

has a modulus and a phase limited by the relationship

$$\frac{T}{T_1} - 2\,\mathrm{th}\left(\frac{T}{2T_1}\right) \leqslant A(x) \leqslant \frac{T}{T_1};\ 0 \leqslant \alpha(x) \leqslant \frac{\pi}{2}. \tag{11}$$

Locus of $\frac{1}{e^{jxN} - 1}$ with $\underline{x}$ varying between $-\pi$ and $+\pi$ consists of N superimposed loci of $\frac{1}{e^{jx} - 1}$, each of which represents a straight line passing through point $-\frac{1}{2}$ parallel to the imaginary axis and converging at $+\infty$.

It is easy to plot the frequency characteristic (10) by using Fig. 3 graphs.

## SUMMARY


The control method by means of a self-adjusting program ensures the solution of a number of specific problems (for instance the problem of stabilizing the hot-rolled sheet thickness), which cannot be solved by any other known automatic control methods.

Above method can be put into practice by means of a simple computer.


## APPENDIX

Let us find an open system transfer function $H^*(q)$. With the condition that Z(p) in (6) has only simple poles and with zero initial conditions, the reaction of the linear part of the system to a unit step input, impressed at instant t = 0, is determined by the transfer function

$$h(t) = c_0 + c_1 t + \sum_{\nu=2}^{l} c_\nu e^{p_\nu t}, \tag{12}$$

where $p_\nu$ represents the poles of Z(p) not equal to zero, and $c_0$, $c_1$, and $c_\nu$ are the coefficients of development of function $\frac{1}{p}$ Z(p) into partial fractions, determined in the usual manner:

$$c_0 = \frac{d}{dp}\left[\frac{P(p)}{Q(p)}\,p\right]_{p=0}, \quad c_1 = \frac{P(0)}{Q'(0)}, \quad c_\nu = \frac{P(p_\nu)}{Q'(p_\nu)\,p_\nu}.$$

The reaction of the linear part of the system to a unit pulse of duration T, impressed at instant t = 0, is determined by a pulse transfer function $h'(t)$:

$$h'(t) = h(t) - h(t - T) = c_1 T + \sum_{\nu=2}^{l} c_\nu e^{p_\nu t}\,(1 - e^{-p_\nu T}).$$

By introducing a dimensionless time $\bar{t} = t/T$, and considering values of $h'(\bar{t})$ at integral instants of time $\underline{n}$ (where n = 1, 2, . . . ) we obtain

$$h'(n) = c_1' + \sum_{\nu=2}^{l} c_\nu \frac{e^{q_\nu} - 1}{e^{q_\nu}}\, e^{q_\nu n}, \tag{13}$$

where

$$q_\nu = p_\nu T,\ c_1' = c_1 T.$$

Let us now find the reaction arising at the outputs of IL when they receive through the switch and the link with operator (p/K) Z(p) a pulse of unit amplitude and duration starting at time t = 0 (Fig. 4).

The instant IL 1 input is closed and the initial conditions established corresponds to t = 0.

It is obvious that the input of the integrating link number $\underline{k}$ will be connected at instant aN + k − 1 and disconnected at the instant aN + k, where a = 0, 1, 2, . . . , ∞, and N is the number of integrating links ($N = T_C/T$).

In the intervals when the IL input remains disconnected, the reaction at the output remains equal to the reaction which existed at the instant the input was connected (Fig. 5).

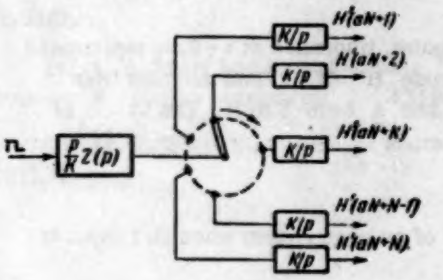

Fig. 4

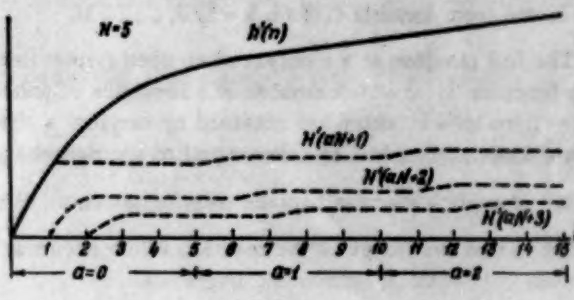

Fig. 5

Let us denote this reaction by H'(aN + k), and evaluate it by using (13) taking into account that all the links of the circuit possess detecting properties:

$$H'(aN+1)=c_1'+\sum_{\nu=2}^{l} c_\nu (e^{q_\nu}-1)+\sum_{j=1}^{a} h'(jN+1)-h'(jN)=$$
$$=c_1'+\sum_{\nu=2}^{l} c_\nu (e^{q_\nu}-1)+\sum_{j=1}^{a}\sum_{\nu=2}^{l} c_\nu \frac{(e^{q_\nu}-1)^2}{e^{q_\nu}} e^{q_\nu N j}=$$
$$=c_1'+\sum_{\nu=2}^{l} c_\nu (e^{q_\nu}-1)+\sum_{\nu=2}^{l} c_\nu \frac{(e^{q_\nu}-1)^2}{e^{q_\nu}} \frac{(1-e^{q_\nu N a})e^{q_\nu N}}{1-e^{q_\nu N}}. \tag{14}$$

Expression $c_1'+\sum_{\nu=2}^{l} c_\nu (e^{q_\nu}-1)$ represents the reaction at the instant n = 1.

Similarly at k ≠ 1 we obtain

$$H'(aN+k)=\sum_{j=0}^{a} h'(jN+k)-h'(jN+k-1). \tag{14a}$$

For the convenience of further operations let us represent (14a) in the form

$$H'(aN+k)=h'(k)-h'(k-1)+\sum_{j=1}^{a} h'(jN+k)-h'(jN+k-1)=$$
$$=\sum_{\nu=2}^{l} c_\nu \left(\frac{e^{q_\nu}-1}{e^{q_\nu}}\right)^2 e^{q_\nu k}+\sum_{j=1}^{a}\sum_{\nu=2}^{l} c_\nu \left(\frac{e^{q_\nu}-1}{e^{q_\nu}}\right)^2 e^{q_\nu (k+jN)}=$$
$$=\sum_{\nu=2}^{l} c_\nu \left(\frac{e^{q_\nu}-1}{e^{q_\nu}}\right)^2 e^{q_\nu k}+\sum_{\nu=2}^{l} c_\nu \left(\frac{e^{q_\nu}-1}{e^{q_\nu}}\right)^2 \frac{(1-e^{q_\nu aN})e^{q_\nu (N+k)}}{1-e^{q_\nu N}}. \tag{15}$$

Now let us take into account the operation of the output switch and the existence of elements with delay $\tau$ in the circuit. As it has already been stated the output and input of each IL are never closed simultaneously. Hence, the value of the output signal received from each IL remains constant during the switching of the output and equal to the reaction in IL formed at the last disconnection of its input. The output switch leads the input only by $\tau = \tau/T$ positions. In considering the cyclic operation of the switches we find the time interval between the instant an IL output is connected and the instant its input was last connected (i.e., the delay in the computer) to be equal to $N-\tau$. The total delay of the system will then be $\tau + (N-\tau) = N$.

Thus, the reaction at the output of an open system formed at the closing of the k-th IL output is represented by a sequence of rectangular pulses which appear at instants $bN+k-1$, and finish at instants $bN+k$, where $b=a+1$, and which have an amplitude $H'(aN+k)$. Quantity $H'(aN+k)$ is determined from formula (14) for $k=1$, and from formula (15) for $k=2, 3, \ldots, N$.

The full reaction at the output of an open system due to a unit pulse, impressed at $t=0$, is represented by step function $H(n)$ which consists of a sequence of pulses of amplitude $H'(aN+k)$ and duration from $bN+k-1$ to $bN+k$ which are obtained by varying $\underline{a}$ from 0 to $\infty$ and $\underline{k}$ from 1 to N. The values of $H(n)$ at discontinuities will be taken equal to the right-hand side values.

Let us apply a discrete Laplace transformation to $H(n)$.

Let us find the image of the reaction which occurs at the output of an open system when IL 1 input is connected:

$$H^*(q,1)=\sum_{b=1}^{\infty}e^{-qNb}H(aN+1)=$$

$$=\sum_{a=0}^{\infty}a^{-q(a+1)N}\left[c_1'+\sum_{\nu=2}^{l}c_\nu(e^{q_\nu}-1)+\sum_{\nu=2}^{l}c_\nu\frac{(e^{q_\nu}-1)^2}{e^{q_\nu}}\frac{(1-e^{q_\nu Na})e^{q_\nu N}}{1-e^{q_\nu N}}\right]$$

$$=c_1'\frac{1}{e^{qN}-1}+\sum_{\nu=2}^{l}c_\nu(e^{q_\nu}-1)\frac{1}{e^{qN}-1}+$$

$$+\sum_{\nu=2}^{l}c_\nu\frac{(e^{q_\nu}-1)^2e^{q_\nu N}}{e^{q_\nu}(1-e^{q_\nu N})}\left(\frac{1}{e^{qN}-1}-\frac{1}{e^{qN}-e^{q_\nu N}}\right). \tag{16}$$

Let us now find the image of the reaction due to connecting to the output of the k-th IL $(k\neq 1)$:

$$H^*(q,k)=\sum_{b=1}^{\infty}e^{-q(Nb+k-1)}H(aN+k)=$$

$$=\sum_{a=0}^{\infty}e^{-q[N(a+1)+k-1]}\left[\sum_{\nu=2}^{l}c_\nu\left(\frac{e^{q_\nu}-1}{e^{q_\nu}}\right)^2e^{q_\nu k}+\sum_{\nu=2}^{l}c_\nu\left(\frac{e^{q_\nu}-1}{e^{q_\nu}}\right)^2e^{q_\nu k}\frac{1-e^{q_\nu aN}}{1-e^{q_\nu N}}e^{q_\nu N}\right]=$$

$$=\sum_{\nu=2}^{l}c_\nu\left(\frac{e^{q_\nu}-1}{e^{q_\nu}}\right)^2\frac{e^{k(q_\nu-q)}e^{q}}{e^{qN}-1}+$$

$$+\sum_{\nu=2}^{l}c_\nu\left(\frac{e^{q_\nu}-1}{e^{q_\nu}}\right)^2\frac{e^{k(q_\nu-q)}e^{q_\nu N}e^{q}}{1-e^{q_\nu N}}\left(\frac{1}{e^{qN}-1}-\frac{1}{e^{qN}-e^{q_\nu N}}\right). \tag{17}$$

On the basis of the linearity theorem, the image of the full reaction at the output will be determined by

$$H^*(q)=D\{H(n)\}=H^*(q,1)+\sum_{k=2}^{N}H^*(q,k). \tag{18}$$

By substituting (16) and (17) in (18) and following the required transformations we obtain Expression (8).

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Received September 5, 1957

# DEVELOPMENT OF AN ALMOST OPTIMAL SYSTEM BY MEANS OF AN ELECTRONIC ANALOG

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The paper deals with certain problems of designing a control element, almost optimal in its speed of operation, for an electric drive of a rolling mill pressure mechanism. The design method consists in dividing the required part of the system into sections, in designing for each of them a simple optimal control element, and regulating the input to the sections in such a way that the difference between its actual and optimum values tends to zero.

The system was tested and the best values for its parameters were selected by means of an electronic analog. The test results show that the system in question decreases the regulation time to 0.5 - 0.4 of that obtained by a linear control system.

## I. Object of Investigation

The main object of an automatic control system for a cold-rolling continuous mill is to ensure the output of a strip with a constant thickness. The main reason for variations in the output thickness is the uneven thickness of the metal to be rolled, i.e., of the strip entering the mill.

The effect of the rolled metal thickness variations which have a large amplitude and small frequency can be eliminated by means of an automatic control system operating the pressure mechanism of the first stand. The block schematic of such a system is shown in Fig. 1. The strip of thickness $H_0$ which enters the mill passes between rolls $B_1$ and $B_1'$ of the first stand, where the strip is rolled down to a thickness of $H_1 < H_0$. At a distance $l$ from the first stand there is a noncontact micrometer M which measures thickness $H_1$. The measurement results are transmitted to regulator P. If the value of $H_1$ deviates from the normal, the regulator operates and displaces the rolls, approaching them if $H_1$ is larger and separating them if $H_1$ is smaller than the normal thickness.

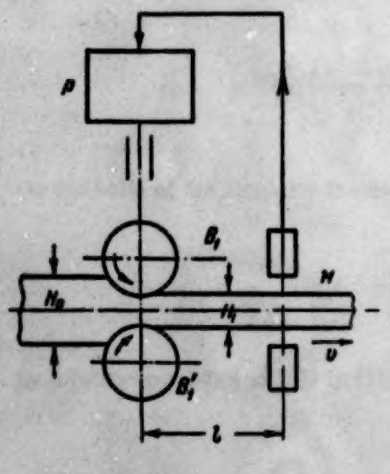


Fig. 1

The lag in measurement due to the distance between the rolls and the micrometer, the inertia of measuring instruments, dry friction torque, control system time constants, backlash in transmission and a small reduction rate slow down the operation of the system.

The operation of the system can be speeded up by using an hydraulic drive which ensures a rapid small displacement of the rolls without any wear in the drive. In existing mills, however, the changeover to hydraulic drives would involve considerable redesigning alterations. It is useful, therefore, to investigate possibilities of a maximum increase in the speed of the system with the existing electric drive by

improved efficiency and an approach to optimum speed conditions. Below we investigate such a possibility on the basis of certain simple considerations and by means of an electronic analog.

## II. Methods of Designing Near-Optimum Systems

The theory of optimum systems [1 and 2] indicates methods for designing control elements for a wide range of applications. However, a synthesis of a strictly optimum system leads to very complicated schemes and labor-consuming calculations. In complex conditions, therefore, the application of this method is not expedient. A strictly optimum system is not usually required; it is only necessary to design a near-optimum system.

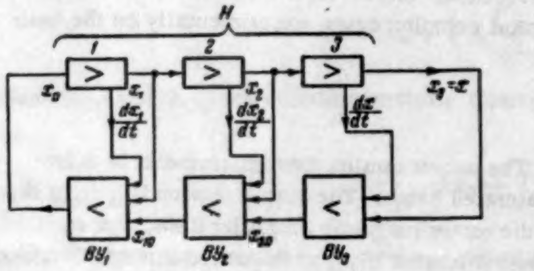

Fig. 2

Another method [2] consists in an approximation of a rigorous formula for a commutation surface in phase space to a simpler expression and subsequent design of a simpler scheme to a simpler formula. It is advisable to use this method when a close approximation to the optimum is required with most diverse initial conditions. Since, in the pressure mechanism drive, typical effects are those of disturbances which are intermittent, sinusoidal and linear with respect to time, only a limited sphere of the initial conditions has the greatest probability of occurring. In a complex system the plotting of a commutation surface requires many labor-consuming calculations. In practice such a plotting can only be done with the help of auxiliary calculating devices, such as automatic and semi-automatic synthesizers; this method was, therefore, not employed either. In addition to these methods, technical literature contains others, not based on the theory of optimum systems, but relying on certain physical ideas for speeding up transitional processes. Such methods are examined, for instance, in papers [3] and [4]. These methods are often very successful in application to systems of a low order, but they cannot always be taken as a reliable foundation for working out complex, near-optimum systems. Supplementing general methods, however, by some specific physical ideas usually leads to good results.

Finally, the last group of methods consists of working out near-optimum systems by using the theory of optimum systems for systems of a lower order and from them constructing those of a higher order. Such a method is examined, by the way, in paper [5].

The process applied below belongs to the latter group of methods. Analog investigations have shown that with the help of this method it is possible to design systems approaching the optimum; in the most complex cases the method should be supplemented by some physical ideas. Thus, this method acquires traits close to the third group of above-mentioned methods.

The underlying idea of this method is very simple. Let us explain it for a particular case when the variable part of the system H (Fig. 2) is, for instance, of the sixth order. It should be noted that this method is applicable to systems with an invariable part of any order. Let, for instance, part H of the system be divided into three sections of the second order 1, 2, and 3 (Fig. 2) (this method is also applicable when parts of the system are of the first or third order). Let us assume that in the system it is possible to measure the output quantities $x_1$, $x_2$, and $x_3 = x$ of these sections, and to obtain even if it were approximately their derivatives $dx_1/dt$, $dx_2/dt$, and $dx_3/dt$. Quantity $x_3 = x$ can be regulated.

Let us concentrate on section 3, part H. By examining it separately, with the condition of limiting the maximum of the $x_2$ modulus and also possibly $dx_2/dt$, it becomes possible to find what should be the ideal variation in quantity $x_2$ in order to ensure an optimum process in this system of the second order. The required ideal value of $x_2$, which we shall denote as $x_{20}$, can be obtained at the output of the computer $BY_3$, which is constructed on the basis of theory [1, 2]. Computer $BY_3$ must incorporate $\underline{x}$ and $dx/dt$ (or, for instance, their linear combinations). If $x_2$ were equal to $x_{20}$ the process in section 3 would be optimal. It will be near-optimum when $x_2$ is close to $x_{20}$, or at least when the absolute value of misalignment $x_2 - x_{20}$ decreases as rapidly as possible.

By examining section 2 we shall see that for a rapid elimination of misalignment $x_2 - x_{20}$ it is necessary to change in a definite manner the input value of $x_1$ of this section, taking into consideration the limitation of

$x_1$ modulus and possibly of $dx_2/dt$. It is known how to design computer $BY_2$ whose output should feed the "ideal" value $x_{20}$ to the input of section 2; $BY_2$ should have $x_2$ and $dx_2/dt$ or, for instance, two of their linear combinations impressed on its input. By repeating the same reasoning we arrive at the design of computer $BY_1$, which should control the input quantity $x_0$ of part H of the system in such a way as to eliminate as rapidly as possible the misalignment $x_1 - x_{10}$. Thus, the problem of designing the control part of a near-optimum system has been reduced to a number of problems of designing optimum systems of a lower order.

This method is applied below in a rather simplified way. Part H of the system is divided into two sections, and computers $BY_1$ and $BY_2$ are designed for the two sections. Moreover, the choice of the computer parameters and some auxiliary circuits is carried out, in the most complex cases, experimentally on the basis of qualitative considerations.

## III. Analog System*

The pressure mechanism schematic is given in Fig. 3. The output tension from micrometer M is impressed on the electronic amplifier EA, which has dead and saturated bands. The output tension $U_{ea}$ from the electronic amplifier is impressed on the input winding $W_1$ of the rotary magnetic amplifier RMA. For stabilizing the RMA, winding $W_2$ is used on which the derivative of the output tension $U_{rma}$ is impressed through transformer Tr. Thus, a flexible feedback is obtained.

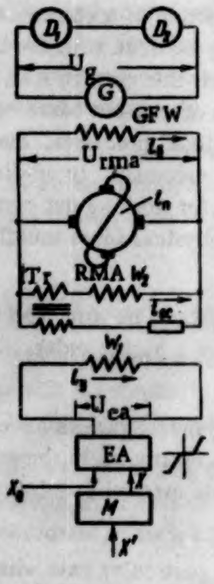


Fig. 3

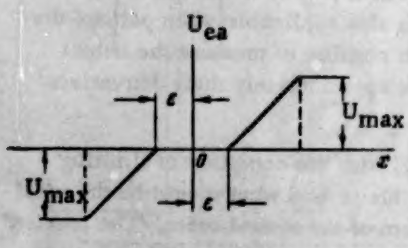


Fig. 4

The output tension of the RMA is impressed on the field winding GFW of generator G which supplies drive motors $D_1$ and $D_2$ (in the circuit these motors can be substituted by a single equivalent one). The equation for the output quantity X of micrometer M has the form:

$$X(t) = X'(t - \tau), \tag{1}$$

where X' is the strip thickness at the stand output, and $\tau = l/v$ is the time lag of measurement.

Quantities $l$ and $v$ are shown in Fig. 1.

The electron amplifier equation has the form:

$$U_{ea} = f(x), \tag{2}$$

where function $f(x)$ is shown on Fig. 4.

Quantity $x$ which is determined by formula

$$x = X_0 - X, \tag{3}$$

is the deviation of X from its nominal value of $X_0$. The equation for the equalizing current $i_y$ in winding $W_1$ takes the form

$$i_y + T_y \frac{\alpha i_y}{dt} = \frac{U_{ea}}{R_y}. \tag{4}$$

Tension $U_s$ in the RMA shunt winding circuit is determined by the expression

$$U_s = k_1 (i_y + i_{oc}), \tag{5}$$

where $i_{oc}$ is the current in the flexible feedback circuit.

The equation for current $i_s$ in the shunt winding circuit takes the form

$$i_s + T_s \frac{di_s}{dt} = \frac{U_s}{R_s} \tag{6}$$

* The basic equation and system data were obtained from A. S. Filatov (also see [9]).

where $T_s$ and $R_s$ are the time constant and the resistance of the shunt winding.

Finally, the RMA output tension is

$$U_{rma} = k_2 i_s, \tag{7}$$

where $k_2$ is a constant. The action of the flexible feedback circuit is expressed by

$$i_{oc} + T_{oc}\frac{di_{oc}}{dt} = T_{oc}\frac{dU_{rma}}{dt}\frac{1}{R_{oc}}, \tag{8}$$

where $T_{OC}$ and $R_{OC}$ are certain constants. Current $i_B$ in the generator field circuit is obtained from equation

$$i_B + T\frac{di_B}{dt} = \frac{U_{rma}}{R_B}, \tag{9}$$

where T and $R_B$ are the time constant and resistance of the field circuit. Neglecting the effects of saturation and hysteresis the relation between the generator open circuit emf and current will be

$$E_g = k_3 i_B, \tag{10}$$

where $k_3$ is a constant.

The equation for the speed of the motor has the form

$$\theta\frac{dn}{dt} + 2n = \frac{E_g}{C_e} - \Delta n_c, \tag{11}$$

where $\theta$ and $C_e$ are constants.

$\Delta n_c$ expresses the effect of dry friction. This quantity should change its sign with that of speed $\underline{n}$. The speed of rolls displacement is

$$v = \frac{nt_p}{60i_H}, \tag{12}$$

where $t_p$ is the screw pitch and $i_H$ the reduction rate.

Finally X' is found from the equation

$$X' + \alpha bX' = \int v\,dt. \tag{13}$$

Quantity X' represents the thickness (more accurately, its variations from a certain constant value) and $\alpha bX'$ the strain in the rolls and stand proportional to X'. Quantity $\alpha b = \text{const}$.

## IV. Electronic Analog of the Pressure Mechanism

The analog block schematic is given in Fig. 5. The model of the basic part of the system is given in the upper part of the schematic. The lower part represents the nonlinear control part which will be examined later.

In the system itself, ignoring the nonlinear control part, the quantity $\underline{x}$, i.e., the deviation of X from its nominal value $X_0$ is impressed straight onto the input of the rotary magnetic amplifier (i.e., in the absence of the nonlinear control part, amplifier 1 output is connected to point $\underline{a}$ in Fig. 5).

The analog input has a summing amplifier 1. Amplifier 4 link represents a nonlinear section* which provides a characteristic of the dead band type. Amplifier links 2 and 3 with lagging elements simulate the

* The circuit for the section was proposed by A. I. Manukhin.

two amplifying stages of the rotary magnetic amplifier. The rotary magnetic amplifier feedback is simulated by element 18 with the operator $K(p) = pT_c/(1 + pT_c)$. With the given parameters this feedback did not substantially effect the transient state of the system, therefore, it was subsequently disconnected. The lagging elements in amplifiers 5 and 6 simulate the generator and motor. The integrating element of amplifier 9 simulates the transformation of motor speed into displacement of the pressure screws and variation of the spacing. The noncontact micrometer is also simulated by the lagging element of amplifier 10. In addition to the links enumerated above, the pressure mechanism analog comprises the following sections and elements: a) a section simulating dry friction, attached to amplifier 7 [6]; b) a backlash section BN-6 [8]; c) a noncontact micrometer measuring delay section BPZ-2 and the lagging element of amplifier 10, which simulates the micrometer system time constant; d) inverting amplifiers 8, 11, and 14 used for reversing the sign.

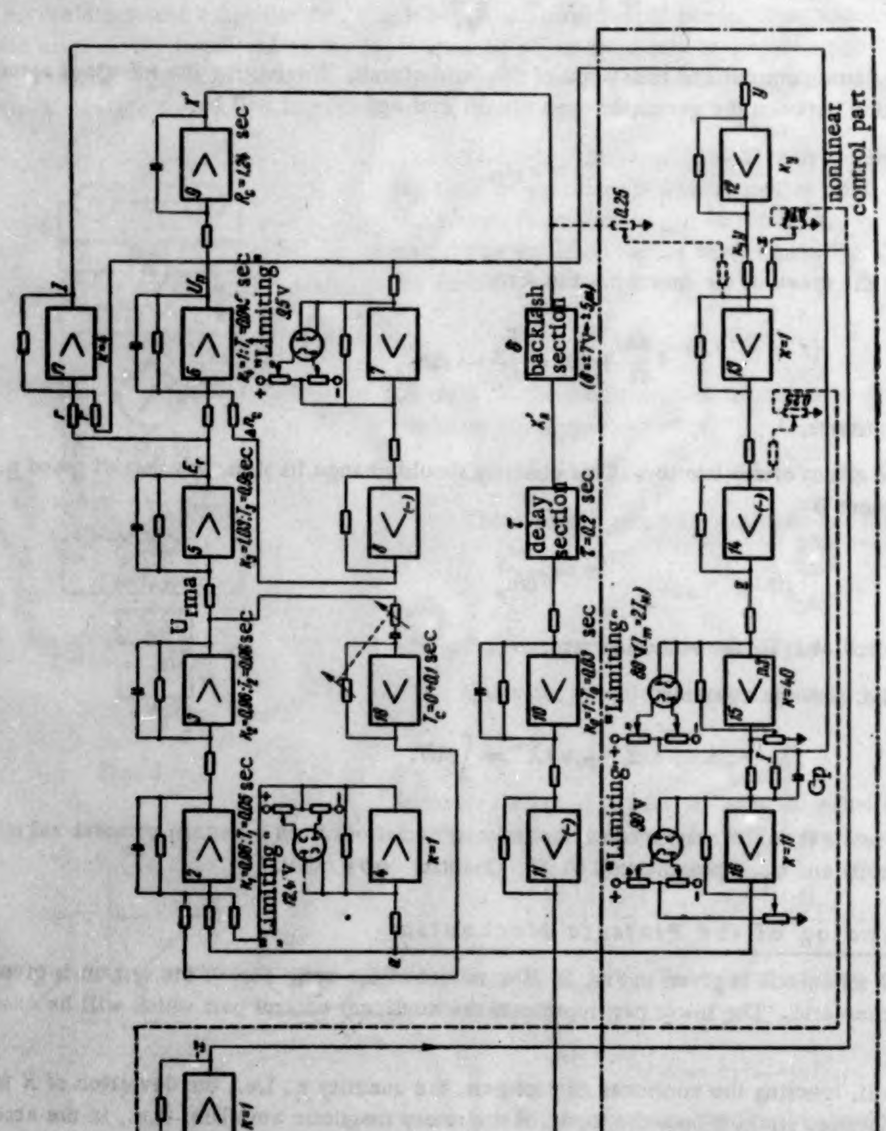


Fig. 5

The value of motor current I is obtained from amplifier 17 according to equation

$$E_g - \lambda n = IR. \tag{14}$$

Where $\lambda n$ is the motor back emf and R is the armature resistance. Hence

$$I = \frac{1}{R}(E_g - \lambda n). \tag{15}$$

Tensions at the output of any analog amplifier vary within the limits of ± 100 v (working range). All the processes are simulated at their natural time scale, hence all the analog time constants have the same values as in the actual system.

The gain of links is determined from the equation

$$K = \frac{U_{out} M_{out}}{U_{in} M_{in}}, \tag{16}$$

where $M_{out}$ and $M_{in}$ are the scales of variables at the output and input of the link.

The scale of a variable is the ratio of the maximum tension at the analog link output to the equivalent value in the actual system.

In accordance with the given system parameters the following scales were selected:

$$M_{uea} = 1, \quad M_{urma} = \frac{100\ v}{250\ v}, \quad M_s = \frac{100}{724}\ \frac{v}{rpm}, \quad M_x = \frac{100}{64.8}\ \frac{v}{\mu},$$

$$M_{eg} = \frac{100\ v}{500\ v}, \quad M_I = \frac{40\ v}{64\ a}.$$

By means of Expression (16) all the link gains were calculated.

The time constant of the integrating link is determined by comparing the equations of the actual link with its analog

$$\frac{d}{dt}\left(\frac{U_x}{M_x}\right) = \xi_0 \frac{U_s}{M_s} \quad \text{и} \quad \frac{dU_x}{dt} = \frac{U_k}{RC}. \tag{17}$$

Whence

$$\frac{1}{RC} = \frac{\xi_0 M_x}{M_s}. \tag{18}$$

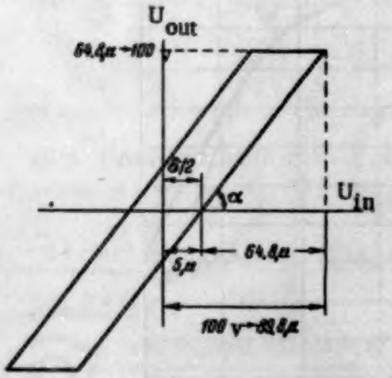


Fig. 6

Quantity $\xi_0$ is determined from the given parameters.

Let the distance covered by the pressure screws be

$$S = \int v\,dt + S_0, \tag{19}$$

the deviation of the thickness from the desired value <u>x</u> is related to S by the equation

$$S = x + \alpha b x. \tag{20}$$

Hence $x = \frac{S}{1 + \alpha b}$ and the displacement $\Delta x = \frac{\Delta S}{1 + \alpha b}$,

i.e.,

$$\Delta x = \frac{1}{1+\alpha b}\int v\,dt = \frac{1}{1+\alpha b}\frac{t_p}{60 i_n}\int n\,dt. \tag{21}$$

Thus,

$$\xi_0 = \frac{1}{1+\alpha b}\frac{t_p}{60 i_n}. \tag{22}$$

Substituting numerical values for the quantities in Equations (22) and (17) we obtain $\xi_0 = 0.073\ \mathrm{sec}^{-1}$ and RC = 1.24 sec.

The backlash link characteristic is shown in Fig. 6. The given backlash is $\delta = 10\ \mu$.

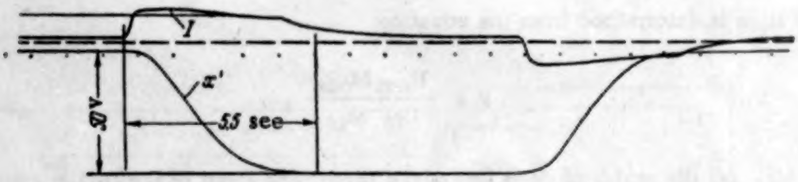


Fig. 7

It is given that $\Delta n_c = 69.3$ rpm. The analog tension corresponding to this figure is $U_{\Delta n_c} = \Delta n_c M_s = 9.5$ v.

Besides the described links of the analog a positive voltage feedback was connected to the motor. This feedback was achieved by impressing part of the motor voltage on the input of the RMA link through an inverting amplifier. This positive feedback was applied in the system (without nonlinear auxiliary coupling) for the purpose of decreasing the transient state duration. Tests have shown that this feedback improves the dynamic characteristic only to a small extent.

## V. Experimental Investigation of a Linear Control System

In this paper only some of the final investigation results of the system are given. In all instances a system with dry friction is considered.

Oscillograms in Figs. 7 and 8 show transient processes in the system when a unit function disturbance is applied to its input. In all the cases the value of the disturbance (of a jump in the steady value) is equal to 50 v which corresponds to a change in the strip thickness of 32.4 $\mu$. Figure 7 shows the transient for a motor voltage feedback system. Figure 8 shows the addition of backlash and delay to the system.

For the system under consideration without feedback frequency characteristics 1) without backlash or delay and 2) with backlash and delay were plotted.

Figure 9 shows the frequency characteristics. In the system with backlash, where the output tension has a nonsinusoidal shape, the gain was calculated for the first harmonic; the shape of the output tension was considered to be trapezoidal.

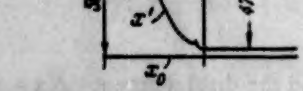


Fig. 8

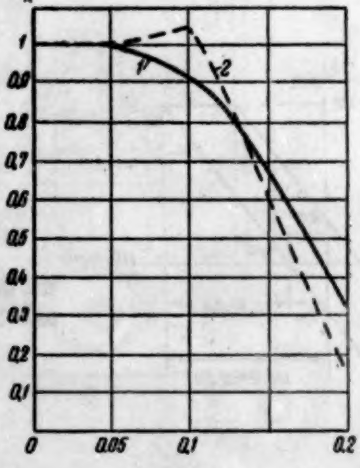


Fig. 9

It will be seen from curves in Fig. 9 that the system works at frequencies not exceeding 0.15 cps; at 0.2 cps the frequency characteristic drops steeply.

## VI. An Optimum System of the Second Order With Dry Friction

As it has been pointed out in section II the design of a near-optimum system can be carried out by means of dividing the fixed part of the system into second-order links. The control part of each link of that type is designed in the form of a relay element, to whose input is connected a linear combination of the link output and its derivative. It remains to be shown that such a control element is near-optimum. This is demonstrated below for a second-order link with dry friction (see the top right-hand side of Fig. 5).

The theory of such a system is described in [7].

Let the equation of the fixed part of the system have the form:

$$J\ddot{x} + f\dot{x} = \pm T_m \pm T_f, \tag{23}$$

where $\pm T_m$ is the turning moment, $\pm T_f$ is the dry friction torque and J and $f$ are constants. During acceleration and braking the resulting torque has the values $T_m - T_f$ and $T_m + T_f$, respectively.

Let us introduce the notation

$$\Omega_1 = \frac{T_m - T_f}{f}, \quad \Omega_2 = \frac{T_m + T_f}{f}, \quad \tau = \frac{J}{f}. \tag{24}$$

Then the phase trajectory equation during braking will assume the form:

$$\Omega_2\tau \ln\left(1 - \frac{\dot{x}}{\Omega_2}\right) + \tau\dot{x} + x = \Omega_2\tau \ln\left(1 - \frac{\dot{x}_2}{\Omega}\right) + \tau\dot{x}_0 + x_0.$$

At a certain instant during braking, $\underline{x}$ and $\dot{x}$ will become equal to zero. This will occur if $x_0$ and $\dot{x}_0$ are related to each other by the expression

$$0 = \Omega_2\tau \ln\left(1 - \frac{\dot{x}_0}{\Omega_2}\right) + x_0 + \tau\dot{x}_0.$$

It follows from the above that an equation for a commutation line in the second quadrant of the phase plane can be obtained from (25) if $x_0$ is substituted by $\underline{x}$ and $\dot{x}_0$ by $\dot{x} = y$. As a result we obtain

$$x = -\tau y - \Omega_2\tau \ln\left(1 - \frac{y}{\Omega_2}\right). \tag{26}$$

The curve in the fourth quadrant is asymmetrical with respect to the one in the second quadrant.

With $y/\Omega_2 \ll 1$, which occurs in practice, Formula (26) acquires a very simple form:

$$x \approx -\tau y, \tag{27}$$

i.e., the commutation line can be considered to be a straight line.

Thus, a near-optimum control part can be attained by impressing on the input of relay link a linear combination $x + \tau y$.

## VII. Experimental Investigation of a System With a Nonlinear Control Part

### 1. A system without backlash or delay.

Figure 5 shows a schematic of an analog system with nonlinear auxiliary connections.

Quantities X and X' differ by the amount of the error in the system:

$$x = X_0 - X = X_0 - X'(t - \tau). \tag{28}$$

Quantity $y = dx/dt$ is obtained at the input of the integrating link. Tension $\underline{y}$ is proportional to the motor speed. We have $z = x + f(y) \approx x + $ $+ \text{const}\ y$.

Sum $\underline{z}$ is impressed on the input of the high-gain amplifier 15 relay link with diode limiters.

The characteristic of this link has the form:

$$I_0 = \begin{cases} I_m & \text{with} \quad z > 0, \\ -I_m & \text{with} \quad z < 0, \end{cases}$$

i.e.,

$$I_0 = \begin{cases} I_m & \text{with} \quad x > f(y), \\ -I_m & \text{with} \quad x < f(y). \end{cases}$$

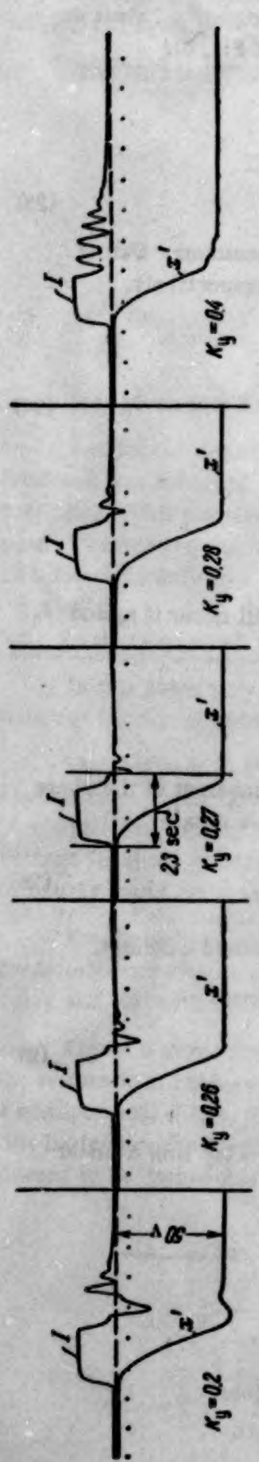

Fig. 10

Function $f(y)$ can be taken as approximately linear (see below).

The next part of the schematic represents an automatic regulator which keeps the drive current equal to $I_0$ which assumes values of $I_m$ or $- I_m$. This system is also near-optimum. The tension at the output of amplifier 16 is also limited; the gain of amplifier 16 must be much greater than unity.

Capacitor $C_p$ is included in the circuit in order to obtain a derivative of the output tension. Thus, the relay link input receives the sum of the output quantity and its derivative with certain coefficients.

The first stage of investigation consisted in testing the system with dry friction, but without backlash or delay. The testing amounted to the choice of the following optimum circuit parameters: 1) the capacity of $C_p$; 2) the value of the dead band of relay link; 3) gain of amplifier 16; 4) coefficient $k_y$ for various values of disturbance.

As the result of testing, the following parameter values were selected: 1) $C_p = 0.02\ \mu f$; 2) relay link amplifier dead band = 1.5 v; 3) amplifier 16 maximum gain $k = 11$.

Oscillograms in Fig. 10 show curves of current I transients and the displacement of X' when a unit function disturbance is applied to a system with nonlinear auxiliary connections but without backlash. The oscillograms show transients for various values of coefficient $k_y$. With too small or too large values of $k_y$ the character of transients in X' greatly deteriorates (see Fig. 10, when $k_y = 0.2$ and $0.4$). As the result of comparing the oscillograms, coefficient $k_y = 0.27$ was chosen. Frequency characteristics for such a system are shown in Fig. 11. The characteristics were obtained with a sinusoidal amplitude of 50 v impressed on the input.

## 2. System with backlash.

The next stage in testing was that of a system with backlash. The backlash section was connected at the output of the integrating amplifier and the output displacement was taken from the output of the backlash section. The backlash range was $\pm 5\ \mu$ which corresponds in the model to $\pm 7$ v. The transient process in the system with backlash greatly deteriorated; the output quantity began to oscillate.

In order to obtain a good shape transient (similar to the one without backlash), the circuit was slightly changed.

Without backlash or delay $X' = X_{meas}$, but with backlash $X'$ can differ considerably from $X_{meas}$. If in addition to $\underline{x}$, $X_0 - X'$ is also applied to the relay link input, the backlash link appears to be shunted. Quantity $\underline{x}$, however, cannot be neglected altogether, since misalignment has to be eliminated by means of this quantity.

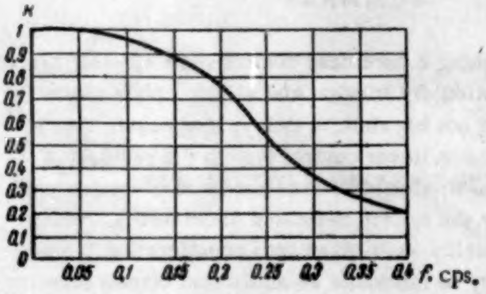


Fig. 11

The circuit is so designed that $\underline{x}$, appearing at the input of the relay link, is slightly reduced but in addition to it quantity $X_0 - X'$ is also impressed on the input.

In Fig. 5 the dotted line shows the additions which had to be made in order to obtain a good transient process. In practice it is possible to obtain the displacement, which in the analog occurs before the backlash link, by means either of low-power transmission with minimum backlash from the motor axle, or of an integrating link with a limiting of misalignment between its output quantity and $X'_b$. In the system with backlash it also became necessary to increase the deadband in the relay link to 2.8 v and raise coefficient $k_y$ to 0.37. Oscillograms of Fig. 12 show the transients for $X'_b$ and current I in a system with nonlinear auxiliary connections and backlash at various values of $X_0 - X'$ (80, 50, and 20 v tensions). $X_0$ and $X'$ are supplied in the same proportion; the proportion of $\underline{x}$ is respectively decreased.

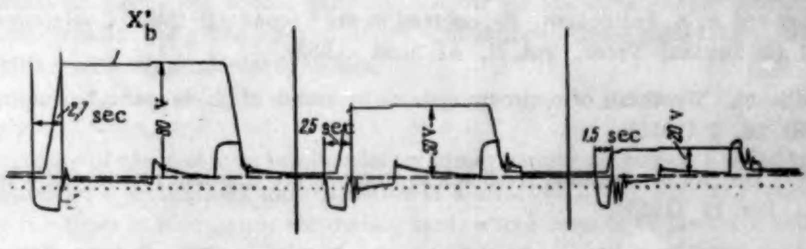


Fig. 12

3. A system with backlash and delay.

The last stage consisted in testing a system with friction, backlash and a constant delay. The delay time was chosen to be $\tau = 0.2$ sec. With the addition of delay, coefficient $k_y$ had to be raised to 0.5.

The oscillogram in Fig. 13 shows transients for $X'_b$ and $X_0 - X_{meas}$ in a system with nonlinear auxiliary connections, backlash and a delay at $k_y = 0.5$, a relay link dead band equal to 2.8 v, $C_p = 0.02$ $\mu$f, amplifier 16 gain of k = 11 and a proportion for $X_0 - X'$ equal to 25%. Frequency characteristics with input tensions of 50 and 20 v were measured for this system.

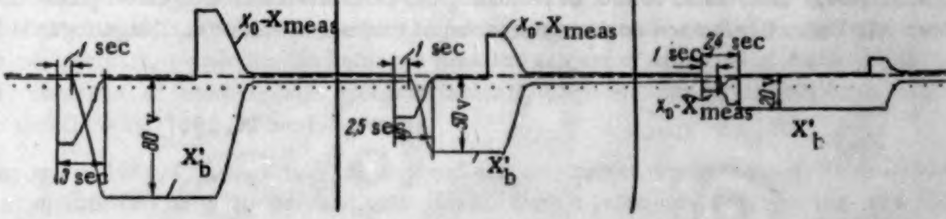


Fig. 13

The calculation of the frequency characteristic was carried out in the same way as before. The shape of the output tension was considered trapezoidal and its first harmonic was taken for calculations. The frequency characteristic is shown in Fig. 14. Curve 1 corresponds to an input amplitude of $x_0 = 50$ v and curve 2 to $x_0 = 20$ v.

In the final circuit (Fig. 5) of the nonlinear control part it is possible to omit amplifiers 12 and 13 summing all the quantities in amplifier 15 and (for the choice of the correct sign) using amplifier 14.

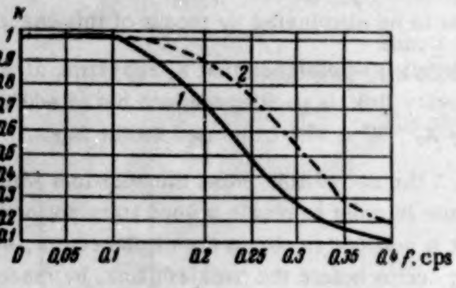


Fig. 14


## SUMMARY

In designing a nonlinear control system, while taking into consideration dry friction and all the orders of the fixed part, but not backlash or delay, it appeared that in comparison with a linear control system the regulation time decreased to about 39% and the band of frequencies transmitted by the system increased about 100%. When backlash and delay were taken into consideration it was found necessary to introduce an additional circuit shunting the backlash. With this arrangement a good transient process was obtained, 2.4 times quicker than for a linear control system, but not as good as for a nonlinear control system without backlash or delay. The nonlinear control part in above instance was sufficiently simple; a similar device can be made for an actual pressure mechanism automatic control system.


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Received June 20, 1957

# MAGNETIC MODULATORS WITH PERPENDICULARLY SUPERPOSED MAGNETIC FIELDS

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The theory and technique of designing magnetic modulators with mutually perpendicular drive and signal magnetic fields are described. Modulators with fundamental frequency and second harmonic outputs are discussed. Experimentally obtained characteristics are given for modulators with ferrocart-2000 cores; these results agree satisfactorily with calculated values.

The low stability of electronic or semiconductor direct current amplifiers has led to a wide use of magnetic modulators for a preliminary transformation of a direct or slowly changing tension into an alternating one. In practice two types of modulators are mainly used, whose cores have the drive and signal magnetic fields either parallel aiding or opposing [1]. The first type is a balanced modulator, for instance, bridge connected.

The output tension frequency of such modulators coincides with that of the drive and its phase changes by 180° with a reversal of the signal polarity. The defect of modulators of this type consists in variations of their balance with changes in the drive voltage, the displacement current or the temperature.

Modulators of the second type are based on the production of even harmonics by the signal. The defect of these modulators consists in the fact that, owing to the impossibility of selecting completely identical cores, the modulator output usually contains large parasitic tension of the fundamental and the third harmonic. Also, the even harmonics of the drive, somewhat attenuated, appear at the modulator output, producing a false signal.

These defects are largely eliminated in magnetic modulators with cores whose drive and signal magnetic fields are mutually perpendicular. The possibility of designing magnetic amplifiers, modulators and similar devices with two mutually perpendicular fields has been the subject of attention of many authors for a long time, and the properties of ferromagnetic bodies in mutually perpendicular fields have been investigated in considerable detail [2-9].

As long ago as 1944, I. L. Bershtein [4] proposed and constructed a magnetically modulated field strength indicator (probe) with two mutually perpendicular fields. Such a probe could also be used as a magnetic modulator, but in this connection it has certain defects, it is very sensitive to external magnetic fields and has a low gain.

In designing a modulator for very weak direct-current signals the following two conditions should be observed:

1) Each of the mutually perpendicular magnetic fluxes must pass only through the magnetic material of the core and must not flow through the air (as in the case of the magnetic probe).

2) The alternating magnetic flux, due to the drive tension, must pass through the same parts of the core as the "direct" flux caused by the signal.

The last condition must be complied with in order to avoid the modulator zero deviation due to hysteresis [1].

In the case of cores made of metallic magnetic materials it was found impossible to preserve both conditions in practice; this delayed the development of such modulators. Only with the appearance of oxide ferromagnetic materials (ferrites) an opportunity arose to make modulators with mutually perpendicular fields, which would satisfy above conditions. Such modulators were proposed in [10].

The present paper deals with the theory and design of modulators with mutually perpendicular fields. Experimental investigation results of such modulators are given.

## Modulator With Frequency Doubling

The frequency doubling modulator core consists of two similar parts containing a ringed groove, which carries the drive winding (Fig. 1). The contacting surfaces of the two halves of the core are polished, in order to decrease the contact resistance for the transverse field. The shape of the core and groove cross section should be selected in such a way as to ensure a constant cross section for the transverse field. The signal $W_y$ and output $W_z$ windings are distributed evenly over the whole length of the core. Currents in these windings induce a longitudinal field. An inductor L (or a blocking filter L, C) is connected in series with the input winding $W_y$, in order to prevent the current second harmonic penetrating into the control circuit.

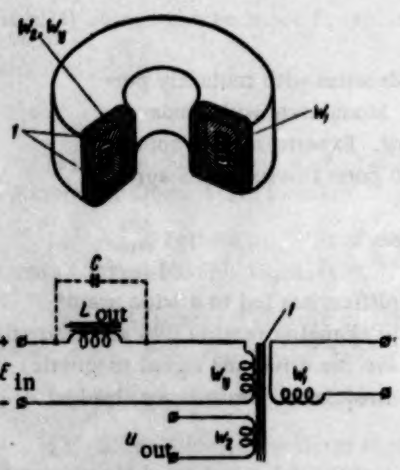

Fig. 1

For convenience of comparing modulators with parallel and mutually perpendicular fields, let us take for the core magnetizing curve the following expression:

$$B = \alpha \operatorname{arc\,tg} \beta H + \gamma H, \tag{1}$$

where $\alpha$, $\beta$, and $\gamma$ are constant coefficients.

Let us also keep in mind that a parallel field modulator has been investigated analytically with the assumption of such an approximation of the magnetizing curve [1].

The magnetizing force due to a signal is

$$H_\parallel = H_y = \frac{0.4\pi W_y I_y}{l}, \tag{2}$$

where $l$ is the mean length of the core.

Let us assume that the transverse magnetizing force obeys a harmonic law:

$$H_\perp = H_m \cos \omega t. \tag{3}$$

Considering the magnetic material to be isotropic we obtain $\frac{B_\parallel}{H_\parallel} = \frac{B_\perp}{H_\perp} = \frac{B}{H}$, where $B = \sqrt{B_\parallel^2 + B_\perp^2}$ and $H = \sqrt{H_\parallel^2 + H_\perp^2}$.

Hence

$$B_\parallel = H_\parallel \frac{B}{H} = H_0 \frac{B_\perp}{H_\perp}. \tag{4}$$

In the absence of a signal, $B_\perp$ is determined by Formula (1) after the substitution of $H = H_\perp$ from (3):

$$B_\perp = \alpha \operatorname{arc\,tg} \beta H_m \cos \omega t + \gamma H_m \cos \omega t. \tag{5}$$

Limited to investigating weak signals only at which $H_0 \ll H_m$, it is possible to consider that $B_\perp$ is determined by Formula (5) even in the presence of signals.

It is known from the theory of ferromagnetic circuits under direct and alternating fields, that if the alternating magnetizing force follows the cosine law (3) the following expression holds for the flux density second harmonic [1]:

$$B_2 = B_{2m} \cos 2\omega t, \quad \text{where} \quad B_{2m} = \frac{1}{\pi} \int_0^{2\pi} B_\parallel \cos 2\omega t d\omega t.$$

By substituting the value of $B_\parallel$ from (4) and taking into consideration (3) and (5) we find:

$$B_{2m} = \frac{\alpha H_0}{\pi H_m} \int_0^{2\pi} \frac{\operatorname{arc\,tg} \beta H_m \cos \omega t}{\cos \omega t} \cos 2\omega t d\omega t + \frac{\gamma H_0}{\pi H_m} \int_0^{2\pi} \cos 2\omega t d\omega t,$$

where the second integral becomes zero.

By introducing the notation $x = \cos \omega t$ and noting that in integrating from 0 to $\pi/2$, $\pi/2$ to $\pi$, $\pi$ to $3\pi/2$ and from $3\pi/2$ to $2\pi$ the first integral has identical values (this can be verified by plotting the expression under the integral against $\omega t$) we obtain:

$$B_{2m} = S_1 + S_2 = \frac{8\alpha H_0}{\pi H_m} \int_0^1 \frac{x \operatorname{arc\,tg} \beta H_m x}{\sqrt{1-x^2}} dx - \frac{4\alpha H_0}{\pi H_m} \int_0^1 \frac{\operatorname{arc\,tg} \beta H_m x}{x\sqrt{1-x^2}} dx.$$

In the last expression the second integral is of a standard form [11] and is equal to

$$S_2 = -\frac{2\alpha H_0}{H_m} \ln [\beta H_m + \sqrt{1 + (\beta H_m)^2}].$$

By integrating by parts we obtain for the first integral

$$S_1 = -\frac{8\alpha H_0}{\pi H_m} \operatorname{arc\,tg} \beta H_m x \sqrt{1-x^2} \Big|_0^1 + \frac{8\alpha\beta H_0}{\pi} \int_0^1 \frac{\sqrt{1-x^2} dx}{1 + (\beta H_m x)^2}.$$

The first term in the right-hand side becomes zero. The expression under the integral of the second term can be represented as a sum of two fractions

$$\frac{\sqrt{1-x^2}}{1 + (\beta H_m x)^2} = \frac{A}{1 + (\beta H_m x)^2} - \frac{B}{\sqrt{1-x^2}},$$

where $A = \dfrac{1 + B + x^2 (B\beta^2 H_m^2 - 1)}{\sqrt{1-x^2}}$, and B is a certain constant.

In the particular case when $B = 1/\beta^2 H_m^2$:

$$S_1 = S_3 + S_4 = \frac{8\alpha\beta H_0}{\pi} \int_0^1 \frac{1 + \frac{1}{\beta^2 H_m^2}}{\sqrt{1-x^2}\,[1 + (\beta H_m x)^2]} dx - \frac{8\alpha\beta H_0}{\pi \beta^2 H_m^2} \int_0^1 \frac{dx}{\sqrt{1-x^2}}.$$

For the second integral $S_4$ of this expression, we have

$$S_4 = -\frac{8\alpha\beta H_0}{\pi \beta^2 H_m^2} \arcsin x \Big|_0^1 = -\frac{4\alpha\beta H_0}{\beta^2 H_m^2}.$$

Let us introduce a new variable into the first integral $S_3$

$$y=\frac{\beta H_m x}{\sqrt{1-x^2}} \quad \text{or} \quad x=\frac{y}{\sqrt{\beta^2 H_m^2+y^2}},$$

After this substitution:

$$S_3=\frac{8\alpha\beta H_0(1+\beta^2H_m^2)}{\pi\beta^3 H_m^3}\int\limits_0^\infty \frac{dy}{1+\frac{\beta^2H_m^2+1}{\beta^2H_m^2}y^2}=$$

$$=\frac{8\alpha\beta H_0\sqrt{1+\beta^2H_m^2}}{\pi\beta^2H_m^2}\operatorname{arctg}\frac{\sqrt{\beta^2H_m^2+1}}{\beta H_m}y\Big|_0^\infty=\frac{4\alpha\beta H_0\sqrt{1+\beta^2H_m^2}}{\beta^2H_m^2}.$$

Thus:

$$B_{2m}=S_1+S_3=S_2+S_3+S_4=\frac{4\alpha\beta H_0\sqrt{1+\beta^2H_m^2}}{\beta^2H_m^2}-\frac{4\alpha\beta H_0}{\beta^2-H_m^2}-$$

$$-\frac{2\alpha\beta H_0}{\beta H_m}\ln\left[\beta H_m+\sqrt{1+\beta^2H_m^2}\right]. \tag{6}$$

For the sake of simplifying notation, let $b_{2m}=\frac{|B_{2m}|}{\alpha}$, $h_0=\beta H_0$ and $h_m=\beta H_m$. Then instead of (6) we shall obtain the following expression for the modulator sensitivity in dimensionless units

$$\frac{b_{2m}}{h_0}=\frac{2}{h_m}\ln\left(h_m+\sqrt{1+h_m^2}\right)-\frac{4}{h_m^2}\left(\sqrt{1+h_m^2}-1\right). \tag{7}$$

The value of the double frequency tension amplitude at the output of the modulator is

$$U_{2m}=2\omega W_2 S\alpha b_{2m}10^{-8}, \tag{8}$$

and the tension at its input is

$$U_y=I_yR_y=\frac{H_0 l}{0.4\pi W_y}R_y=\frac{R_y l h_0}{0.4\pi W_y\beta},$$

where S and $l$ are the cross section area and mean length of the core, respectively, for the longitudinal flux, and $R_y$ is control circuit resistance (that of the control winding and filter inductor or additional resistance).

For the voltage gain we have

$$K_U=\frac{U_{2m}}{U_y}=\frac{16fW_2W_yS}{R_y e\cdot 10^8}\alpha\beta\frac{b_{2m}}{h_0}. \tag{9}$$

Thus, if the magnetizing curve of the core is known [i.e., the values of $\alpha$ and $\beta$ in Formula (1)], the voltage gain of the modulator can be obtained from (9). For this purpose it is necessary to determine beforehand for a chosen value of $h_m$ quantity $b_{2m}/h_0$ from Formula (7) or the graph, shown in Fig. 2 (the continuous line), where the relation between $b_{2m}/h_0$ and $h_m$, as calculated from Formula (7), is given.

It should be stressed that the Formulas (6), (7), and (9) and the characteristics in Fig. 2 apply to the open circuit modulator operation, when the resistance of the load considerably exceeds the internal resistance of the modulator.

Let us note that in the case of parallel fields we have [1]

$$\left(\frac{b_{2m}}{h_0}\right)_{\parallel}=\frac{4}{h_m^2}\left(\frac{1+\frac{h_m^2}{2}}{\sqrt{1+h_m^2}}-1\right). \tag{10}$$

This relationship is shown in a dotted line in Fig. 2.

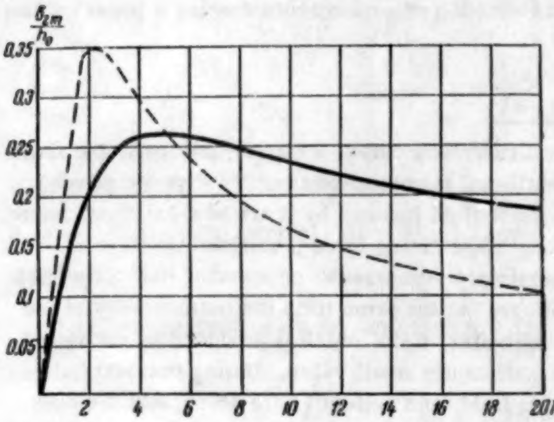

Fig. 2

It will be seen that the maximum sensitivity of the parallel fields modulator in dimensionless units is some 20% larger than that of the modulator with mutually perpendicular fields. However, the quantity $b_{2m}/h_0$ for parallel fields is considerably more dependent on the value of the driving field $h_m$ than for mutually perpendicular fields. Therefore, the second harmonic tension for modulators with mutually perpendicular fields with a correct choice of the value of the driving field will depend much less on the variations of the driving field. At $h_m > 5$ the dimensionless sensitivity of a modulator with mutually perpendicular fields begins to exceed that of a parallel fields modulator. It should, however, be taken into account that the actual sensitivity of the modulator determined as the ratio $B_{2m}/H_0$ is equal to

$$B_{2m} / H_0 = \alpha\beta b_{2m} / h_0.$$

Since it is possible to use, for modulators with parallel fields, alloys with a high permeability such as permalloy, whose factor $\alpha\beta$ is considerably greater than that of ferrite cores, the actual sensitivity and gain of modulators with parallel fields are considerably greater than those of modulators with mutually perpendicular fields. For cores made of the alloy 79NMA, for instance, a value of $\alpha\beta = 33{,}000$ gauss/ergs was obtained, whereas for ferrocart-2000 it is $\alpha\beta = 7{,}700$ gauss/ergs.

The time constant of the control circuit is determined by the following formula:

$$\tau = \frac{L \quad + L_y}{R_y},$$

where $R_y$ is the active resistance of the circuit, L the inductance of the series choke (or filter) and $L_y$ the inductance of the control winding, which is equal to

$$L_y = \frac{0.4\pi \cdot W_y^2 S}{l \cdot 10^8} \frac{B_0}{H_0}.$$

For $B_0$ we obtain from table [11]

$$B_0 = \frac{1}{2\pi} \int_0^{2\pi} B_{\parallel} d\omega t = \frac{H_0}{2\pi} \int_0^{2\pi} \frac{\alpha \operatorname{arc\,tg} \beta H_m \cos \omega t d\omega t}{H_m \cos \omega t} + \gamma H_0 =$$
$$= \frac{\alpha H_0}{2H_m} \ln (\beta H_m + \sqrt{1 + \beta^2 H_m^2} + \gamma H_0).$$

Denoting $K = L/L_y$, we obtain

$$\tau = (1 + k) \frac{L_y}{R_y} = \frac{0.2\pi (1 + k) W_y^2 S}{l R_y \cdot 10^8} \frac{\alpha\beta}{h_m} \left[\ln (h_m + \sqrt{1 + h_m^2}) + \frac{\gamma}{\alpha,\beta}\right]. \quad (11)$$

It should be noted that (as a rule) the following inequalities hold

$$\gamma \ll \alpha\beta \text{ и } \frac{\gamma}{\alpha\beta} \ll \frac{\ln 2h_m}{h_m}.$$

For ferrocart-2000 it was possible, for instance, to make $\gamma = 0$. It is therefore possible to neglect the term containing $\gamma$ in (11).

From Formulas (9) and (11) it becomes clear that with a given value of $h_m$ the voltage gain and time constant of the modulator vary in direct proportion to $\alpha\beta$, which has the dimensions of permeability. Hence, the use of ferrites with a higher permeability in order to increase the gain will inevitably lead to a rise in $\tau$. With a rise in $h_m$ the ratio $K_U/\tau$ slowly rises and in the limit (when $h_m \to \infty$) reaches a value 6 times greater than $K_U/\tau$, which corresponds to $h_m = 5$.

## Modulator With A Fundamental Frequency Output

The core and windings of a modulator with a fundamental frequency output are constructionally the same as for a frequency doubler modulator (Fig. 1). A half-wave rectifier B is connected in series with the driver winding $W_1$ (Fig. 3). Hence, the driver current $i_1$ and the transverse field induced by it are of a pulsating nature.

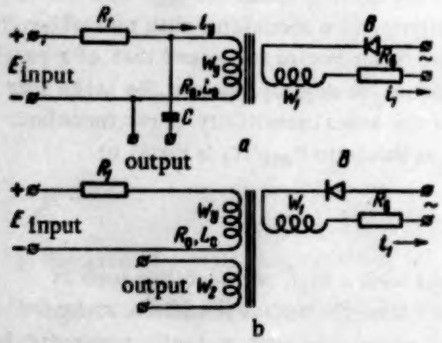

Fig. 3

When the driving magnetizing force is sufficiently large it practically completely saturates the core during half a cycle of the driving voltage. At the same time the permeability of the core for the longitudinal field, which is produced by the signal, drops to an insignificantly small value. During the next half-cycle the driving field (and current i) are absent and the core permeability rises to its original value of $\mu_0$. Thus the driving field (current $i_1$) periodically changes the induction of winding $W_y$ which is connected in the modulated signal circuit. In series with this winding there is connected an active resistance $R_1$ which can include the internal resistance of the signal source E. $R_0$ accounts for the active resistance of winding $W_y$. The output (modulated) tension is tapped off the input winding $W_y$ terminals through capacitor C, which blocks the direct tension of the signal (Fig. 3, a). The output tension can also be taken from another winding $W_2$ of the same transformer $W_y$ (Fig. 3, b).

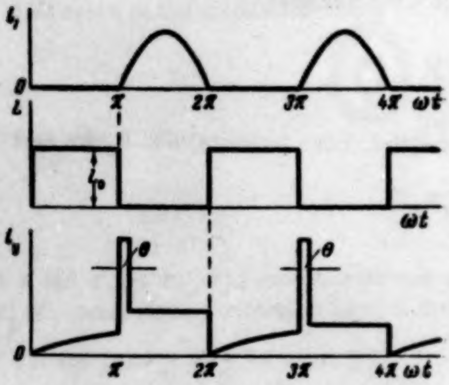
Fig. 4

It should be noted that instead of the active resistance $R_1$ an inductance can be connected to the input of the modulator. Then, however, one of the basic advantages of the modulator, its nonreactive nature, will be lost.

Figure 4 shows schematically the changes due to driving current $i_1$ in inductance L of winding $W_y$ and current $i_y$ through it, when an active resistance $R_1$ is connected in series with the winding. It is assumed that inductance L changes almost instantaneously from its maximum value of $L_0$ to its minimum and vice-versa. The effect of the winding capacity is neglected.

At the beginning of the first half-cycle (t = 0) the inductance jumps to value $L_0$. Since the electromagnetic energy concentrated in the coil cannot change instantaneously, current $i_y$ at time t = 0 must drop to zero. Next, current $i_y$ begins to grow exponentially

$$i_y = \frac{E}{R_y}\left(1 - e^{-\frac{t}{\tau}}\right) \qquad (0 \leqslant \omega t \leqslant \pi),$$

where $R_y = R_0 + R_1$ is the active resistance of the control circuit and

$$\tau = \frac{L_0}{R_y} = \frac{0.4\pi W_y^2 S}{R_y l \cdot 10^8}\,\mu_0. \qquad (12)$$

Bridge measurements of $L_0$ at 1000 cps with various cores and a transverse drive showed that the initial value of the core permeability should be inserted into this formula.

At the end of the first half-cycle the longitudinal component of the flux in the core attains the value of

$$\Phi_\pi = \frac{L_0 i_y(\pi)}{W_y \cdot 10^{-8}} - \frac{E\tau}{W_y \cdot 10^{-8}}(1 - e^{-\frac{\pi}{\omega\tau}}).$$

During a part of the following half-cycle in the output circuit there is a tension of

$$u = E + W_y \cdot 10^{-8}\frac{d\Phi_\parallel}{dt}.$$

The second component of the tension is controlled by the fact that under the influence of the transverse field the longitudinal flux $\Phi_\parallel$ decreases from $\Phi_\pi$ to zero. Let us assume for simplicity that the variation of $\Phi_\parallel$ is linear in the interval $\Delta t = \theta/\omega$. Then $d\Phi_\parallel/dt = \Phi\pi/\Delta t$ $\quad u = E + \frac{\omega\tau E}{\theta}(1 - e^{-\frac{\pi}{\omega\tau}})$.

$\pi \leqslant \omega t \leqslant \pi + \theta$. When $\pi + \theta \leqslant \omega t \leqslant 2\pi$ tension u = E.

For the modulator output tension during the first half-cycle we obtain

$$u_H = i_y R_1 = E\frac{R_1}{R_0 + R_1}(1 - e^{-\frac{t}{\tau}}).$$

Since we assume that from the instant $\omega t = \pi$ the induction of winding $W_y$ becomes very small and cannot limit the value of current $i_y$, the output tension for the next half-cycle will be $u_H = uR_1/(R_0 + R_1)$.

The amplitude of the output tension first harmonic is

$$U_{1m} = \sqrt{U_{1a}^2 + U_{1б}^2}, \tag{13}$$

where

$$U_{1a} = \frac{1}{\pi}\int_0^{2\pi} u_H \sin\omega t d\omega t \qquad U_{1б} = \frac{1}{\pi}\int_0^{2\pi} u_H \cos\omega t d\omega t.$$

After integration we obtain

$$\begin{aligned} U_{1a} &= -\frac{ER_1}{\pi(R_0+R_1)}\left[\frac{\omega^2\tau^2}{1+\omega^2\tau^2}(1+e^{-\frac{\pi}{\omega\tau}}) + \frac{\omega\tau}{\theta}(1-\cos\theta)(1-e^{-\frac{\pi}{\omega\tau}})\right], \\ U_{1б} &= -\frac{ER_1}{\pi(R_0+R_1)}\left[\frac{\omega\tau}{1+\omega^2\tau^2}(1+e^{-\frac{\pi}{\omega\tau}}) + \frac{\sin\theta}{\theta}\omega\tau(1-e^{-\frac{\pi}{\phi\tau}})\right]. \end{aligned} \tag{14}$$

Tension $U_{1m}$ is monotonic and increases with $\tau$. In the limit with $\tau \to \infty$, we have

$$U_{1m} = \frac{ER_1}{R_0 + R_1}\sqrt{\left(\frac{2}{\pi} + \frac{1-\cos\theta}{\theta}\right)^2 + \left(\frac{\sin\theta}{\theta}\right)^2}. \tag{15}$$

Let us note that with $\omega\tau = 10$, tension $U_{1m}$ already reaches 90% of its maximum value, determined by Formula (15).

The variation of $\theta$ within wide limits has but a small effect on the value of the output tension first harmonic. For instance, with $\tau \to \infty$, we find from (15) that the variation of $\theta$ between the limits of 0 and $\pi$ changes $U_{1m}$ by only ± 10% of a certain mean value. With small values of $\theta$ which correspond to large driving fields it is possible to assume that $(1 - \cos\theta)/\theta \approx 0$ and $\sin\theta/\theta \approx 1$. In this case we obtain from (13) and (14)

$$U_{1m} = \frac{ER_1\omega\tau}{\pi(R_0 + R_1)\sqrt{1+\omega^2\tau^2}}\sqrt{4 + \omega^2\tau^2(1 - e^{-\frac{\pi}{\omega\tau}})^2}. \tag{16}$$

With $\omega\tau \to \infty$ we obtain from (16)

$$U_{1m} = 1.19 \frac{R_1}{R_1 + R_0} E. \tag{17}$$

In the case when the load resistance considerably exceeds the control circuit resistance $R_y$ it is possible to decrease $U_{1m}$ by using the transformer connection shown in Fig. 3, b.

It should be expected that the actual values of $U_{1m}$ will be somewhat smaller than those calculated from Formula (13) - (16) mainly due to the fact that the core is not completely saturated during the whole driving half-cycle. The winding capacity affects mainly the higher harmonics.

## Experimental Investigation of Modulators With Mutually Perpendicular Fields

The object of the investigations was to compare the actual and calculated values of gain and the transfer constant for modulators of various types, as well as to establish their behavior in the field. For the experimental modulators, cores of the following dimensions were used:

| Type of core | External diameter, mm | Internal diameter, mm | Height of one-half of the core, mm | Cross section for the longitudinal flux, $cm^2$ |
|---|---|---|---|---|
| 1 | 30 | 18 | 7 | 0.68 |
| 2 | 36 | 24 | 7 | 0.72 |

A typical (mean) magnetization curve, obtained experimentally for a core of the first type, is given in Fig. 5. This curve is a mean between the ascending and descending maximum hysteresis loops. Analytically it can be expressed with sufficient accuracy by the formula $B = 1700 \arctan 32H$ which corresponds to the dotted line in Fig. 5.

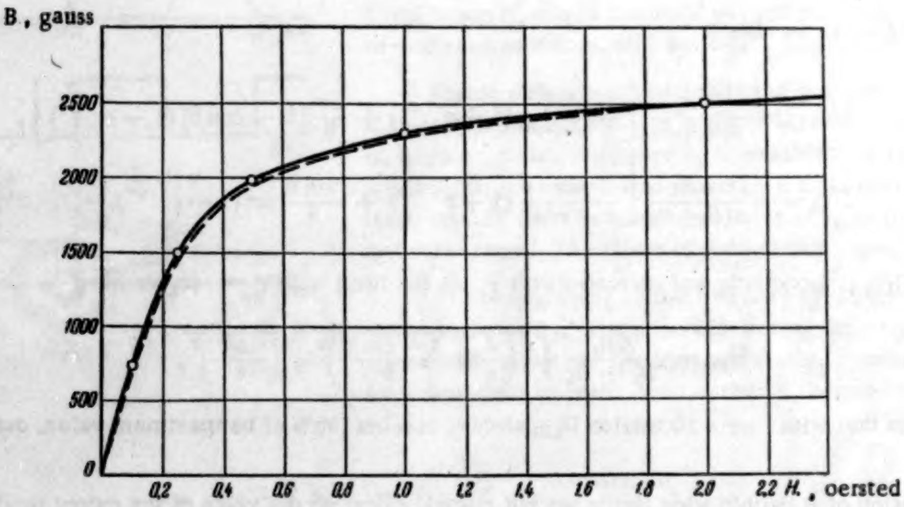


Fig. 5

Test results of a modulator, made with this core, with the following winding data are given below: $W_1 =$ $= 200$ turns, $W_2 = 3200$ turns, and $R_0 = 320$ ohms.

Figure 6 shows the experimental 3 and calculated 1 characteristic of $b_{2m}/h_0 = \varphi(h_m)$ for a frequency doubling modulator, made according to Fig. 1. The experimental characteristic turned out to be shifted along the X-axis as compared with the calculated one. This difference can be due to the fact that in calculating, the existence of a joint (air gap) between the two halves of the core was not taken into account, yet to overcome it an additional magnetizing force is required.

In order to obtain the required transverse field intensity in the core magnetic material with the presence of two air gaps of width $\delta$, the drive current must create a MF with $F = H_m l_\perp + 2\delta B_m$, where $l_\perp$ is the mean length of the magnetic path for the transverse field. (MF — short for magnetizing force).

Denoting by $F / l_\perp = H'_m$ the apparent intensity of the transverse field and substituting $B_m$ by $H_m$ in accordance with Formula (1), we obtain

$$H'_m = H_m\left(1 - \frac{2\delta}{l_\perp}\gamma\right) + \frac{2\delta}{l_\perp}\alpha \operatorname{arc\,tg} \beta H_m$$

or in relative units

$$h'_m = h_m\left(1 + \frac{2\delta}{l_\perp}\gamma\right) + \frac{2\delta}{l_\perp}\alpha\beta \operatorname{arc\,tg} h_m.$$

Dotted curve 2 in Fig. 6 represents the theoretical relationship between $b_{2m}/h_0 = \varphi(h'_m)$ calculated according to Formula (18) for an air gap $\delta = 2\mu$. In this instance the theoretical and experimental curves practically coincide.

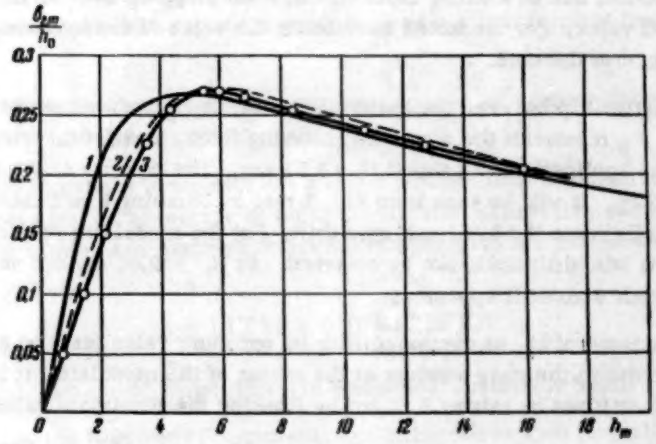


Fig. 6

The maximum voltage gain measured at a drive frequency of f = 500 cps and a current in the control winding of 0.92 $\mu$a was equal to 160. The maximum gain calculated by Formula (9) is 165.

Irregularities of the core cross section for the transverse field and the irregularities of the air gap sizes along the core length lead to the appearance of a stray coupling between the drive and the output windings. This coupling results in the appearance at the output winding terminals, without a signal at the input, of a tension whose harmonics are the same as those of the drive tension. The interrelation of these harmonics was approximately the same as for those of the drive tension. It should be noted that there can also exist a capacitative coupling between the windings.

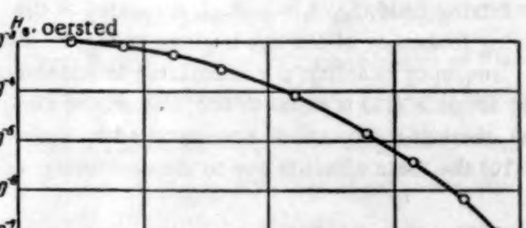


Fig. 7

With a 0.5% harmonic content in the driving tension, the stray second harmonic at the output of the tested modulators was on an average equivalent to an input signal of the order of $10^{-14}$ w. It is obvious that the suppression of the second harmonic by means of a filter would considerably lower the minimum value of a detectable signal. It should be noted that under similar conditions the stray second harmonic at the output of the modulator with parallel fields is usually equivalent to an input signal of the order of $10^{-12}$ to $10^{-10}$ w.

Variations of the ambient temperature by ± 20° C and the drive current by ± 10 % from the nominal value (which corresponded to the maximum gain) did not cause any zero drift or an increase in the stray disturbing tensions at the modulator output. The use of manganese ferrites widened the temperature range to − 50° to + 70° C.

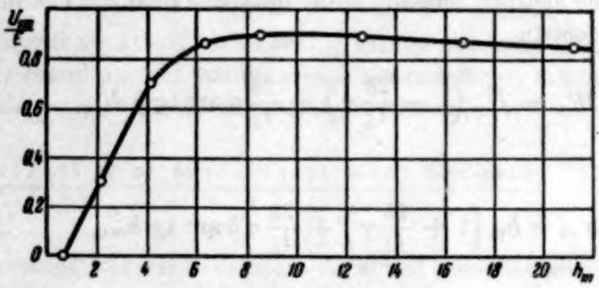


Fig. 8

The most important reason for zero drift in the modulator is hysteresis, which prevents the doubled frequency modulator output tension, due to a strong input signal, from dropping back to its original (zero) value, and keeps it at some residual value. For the tested modulators the value of the hysteresis zero drift varied with the value of the coercive force of the core.

Figure 7 shows the relationship between the hysteresis zero drift $H_B$ and driving field $h_m$ for modulators with cores of the first type. $H_B$ represents the direct magnetizing force, required to bring the output modulator tension back to zero after the application of a signal $H_0 = 0.01$ erg. The polarity of the return field $H_B$ is always opposite to that of the signal $H_0$. It will be seen from Fig. 7 that by choosing a sufficiently large value for $h_m$ it is possible practically to eliminate the hysteresis zero drift. For the modulator in question at $H_0 \leq 0.005$ erg and $h_m = 5$ to 6 the hysteresis zero drift could not be observed. At $H_0 > 0.01$ erg the value of $h_m$ increased and a corresponding small hysteresis zero drift appeared.

Considering that an increase of $h_m$ as compared with its optimum value leads to a rise in the driving power consumed and hence an increase in the stray tensions at the output of the modulator, it is usually expedient to eliminate the hysteresis zero drift not by raising $h_m$, but by limiting the maximum value of the input signal.

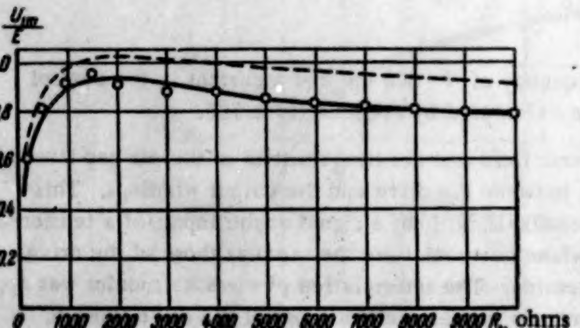


Fig. 9

The hysteresis zero drift depends to a considerable extent on the regularity of the core cross section for the transverse driving field. With an irregular cross section certain parts of the core are only weakly magnetized by the transverse field which is favorable to the appearance of a considerable hysteresis zero drift.

The relationship between the phase shift in a modulator with a fundamental frequency output and the driving field $U_m/E = \varphi(h_m)$, measured at the driving frequency of 500 cps is given in Fig. 8. For this frequency quantity $\omega\tau$ calculated in accordance with formula (12) is equal to 65. Hence, the design value for $U_m/E$ can be determined from (15). With a rising $h_m$ there is a drop in $\theta$ accompanied by a slow decrease in the ratio $U_m/E$. In the region of small $h_m$ ($h_m < 10$) the main effect is due to the core being insufficiently saturated during the driving half-cycle.

The calculated value of $U_{1m}/E$ with $\theta \to 0$, obtained from (15) or (17), is equal to 0.9. The measured value of $U_{1m}/E$ for $h_m = 22$ is equal to 0.84, which is in good agreement with the calculation.

Figure 9 shows the relationship between the transfer constant and resistance $R_1$ at a drive frequency of 500 cps for a modulator with $W_y = 2600$ turns and $R_0 = 160$ ohms. The theoretical characteristic calculated from Formulas (12) and (16) is shown in a dotted line.

The theoretical and experimental curves show a similar relationship between $U_{1m}/E$ and $R_1$. The measured values of $U_{1m}/E$, however, are some 10-15% below the calculated ones.

Investigation of transient processes in the modulator with a fundamental frequency output confirmed that its lag does not exceed half a cycle of the drive frequency.

A modulator with a fundamental frequency output was installed in an electronic recording potentiometer type EPP-09 instead of the normal vibration (contact) chopper. Test working of the potentiometer with the magnetic modulator has shown that the latter ensures the required accuracy of operation.

## SUMMARY


1. Modulators with mutually perpendicular fields have advantages over those with parallel fields in the fact that with a simple construction they lower substantially stray interferring tensions and have more stable characteristics. Moreover, the application of mutually perpendicular fields in modulators with output frequency doubling often ensures the possibility of working without the cumbersome filters in the driving and output circuits, which are required for modulators with parallel fields.

2. The approximation of the core magnetizing curve to an arc tangent permits one to calculate with sufficient accuracy the basic characteristics of the modulator with a fundamental frequency output under no-load conditions.

3. The modulator with a fundamental frequency output works as a transducer and does not, as distinct from the frequency doubling modulator, give any gain.

4. The fundamental frequency modulator has a lower bottom limit of sensitivity than the frequency doubling modulator. It has also the advantage of being practically nonreactive and having a transfer constant independent of the driving tension and ambient temperature when they are varied over fairly wide limits.


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Received November 11, 1957

# EXTERNAL DIMENSIONS OF ELECTROMAGNETIC COMPONENTS

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**The paper deals with the basic relationships between the working efficiency of an electromagnetic system and its magnetic, electric and thermal parameters and the length of its life.**

One of the basic tendencies in modern instrument and electrical appliances industries is a reduction in size and weight.

Let us examine the relation between the parameters and dimensions of electromagnetic components.

The value of the apparent power for a choke (or a transformer under no-load condition) is $P_{\sim} = U_m I_m$. With $R\omega \ll L$

$$P_{\sim} = U_m I_m \approx \omega W S_m B_m \cdot 10^{-8} I_m, \tag{1}$$

or, since

$$H_m = \frac{0.4\pi W I_m}{l_m},$$

$$P_{\sim} = 0.4\pi 10^{-8} \frac{\mu S_m}{l_m} \left(\frac{W I_m}{\sqrt{2}}\right)^2 \omega.$$

Where $U_m$ and $I_m$ are the tension and current amplitudes, L is the inductance, R the resistance of the choke winding, $H_m$ the amplitude of the magnetizing force, $S_m$ the cross section area, $\mu$ the permeability, $l_m$ the length of the magnetic path, and $B_m$ the amplitude of the flux density.

On the other hand

$$\left(\frac{W I_m}{\sqrt{2}}\right)^2 = \frac{I_m^2}{2} R \frac{W^2}{R} = P_L \frac{W^2}{\frac{\rho l_{av} W}{q}} = P_L \frac{Q f_0}{\rho l_{av}},$$

where $P_L$ is the power lost in the windings, Q the cross section area of the winding groove, $f_0$ the stacking factor, $\rho$ the resistivity of winding conductors, $q$ the conductor cross section area, $l_{av}$ the mean length of a winding turn, and R the winding resistance.

Moreover the value of winding losses is related to the winding temperature $\theta_x^\circ$ by the expression

$$0.24 P_L = k S_{cool} (\theta_x^\circ - \theta_0^\circ), \tag{2}$$

where $k$ is the total heat-loss coefficient, $S_{cool}$ is the surface of cooling in cm$^2$, $\theta_0^\circ$ the temperature of the cooling medium, or by

$$P_L = \frac{k S_{cool}}{0.24} (\theta_x^\circ - \theta_0^\circ).$$

Let us introduce notations: $\frac{\mu S_m}{l_m} = G_m$ is the permeance of the magnetic path, $\frac{Q f_0}{\rho l_{av}} = G_e$ is the conductance of the winding volume, $\frac{k}{0.24} S_{cool} = k' S_{cool} = G_\theta$ is the thermal conductivity of the heat flow, from the winding to the surrounding medium. From this we obtain the following expression for the apparent power of the choke:

$$P_\sim = 0.8\pi 10^{-8}\omega(\theta_x^\circ - \theta_0^\circ) G_m G_e G_\theta.$$

A decrease in the core size leads to a decrease in $G_e$ and $G_m$. The latter can be compensated by choosing a material with a large $\mu$. $S_{cool}$ and hence $G_\theta$ also decrease. In order to preserve $G_\theta$ it is necessary to raise the total heat-loss coefficient k', which is achieved either by blowing air or by placing the core in hydrogen, oil or other media. It should be noted that with decreasing pressure of the surrounding gas the total heat-loss coefficient also decreases.

The relation between resistivity and temperature is expressed by the formula

$$\rho = \rho_0 [1 + \alpha_\rho (\theta_x^\circ - \theta_0^\circ)], \tag{3}$$

where $\rho_0$ is the resistivity at $\theta_0^\circ$, and $\alpha_\rho$ the resistance temperature coefficient.

The values of $\mu$ and k' also depend on temperature. For determining $\mu = f(\theta_x)$ it is possible to take the relationship $\mu = \mu_0 [1 - \alpha_\mu (\theta^\circ - \theta_0^\circ)]$, where $\mu_0$ is the permeability at $\theta^\circ = \theta_0^\circ$, and $\alpha_\mu$ the temperature coefficient within the limits of $\alpha_\mu = (1.0 \text{ to } 2.8) \cdot 10^{-4}$ degree$^{-1}$ C. Relationship $k' = f_1(\theta_x)$ can be represented approximately by

$$k' = k_0' [1 + \alpha_k (\theta_x^\circ + \theta_0^\circ)], \tag{4}$$

where $k_0'$ is the total heat-loss coefficient at $\theta_0^\circ$ and $\alpha_k$ the temperature coefficient.

Considering that

$$k_0' \approx 1.2 \cdot 10^{-3} \quad w/cm^2\,^\circ C,\ \alpha_k \approx 6.5 \cdot 10^{-6} \quad w\,(cm\,^\circ C)^2,$$

we obtain the expression

$$k' = 1.2\,[1 + 0{,}0065\,(\theta_x - \theta_0)]\,10^{-3} \quad w/cm^2\,^\circ C,$$

and

$$P \approx 0.4\pi \cdot 10^{-8}\omega(\theta_x^\circ - \theta_0^\circ) G_{m0} G_{e0} G_{\theta 0} f(\theta_x) \frac{1 + \alpha_n (\theta_x^\circ - \theta_0^\circ)}{1 + \alpha_\rho (\theta_x^\circ - \theta_0^\circ)}.$$

Let us also examine the relationship between the external dimensions, consumed power and the mechanical work performed (the "working efficiency") of relays, traction electromagnets, and clutches. The working efficiency of magnetic system $A_e$ with a traction effort $P_e$ and a change in armature travel $\delta$ from $\delta_0$ to $\delta'$ conforms to expression

$$A_e = \int_{\delta_0}^{\delta'} P_e d\delta = \int_{G_0}^{G_B'} \frac{(0.4\pi IW_B)^2}{8\pi} dG \ \text{erg.}$$

Neglecting leakage flux (i.e., $\Phi_s \approx 0$), we can write:

$$IW_B = \frac{R_{mB}}{R_{mB} + R_{mc}} IW = \frac{IW}{\frac{G_B}{G_m} + 1}, \tag{5}$$

where IW are the winding ampere-turns, $IW_B$ the ampere-turns required to carry the flux across the working airgap, $R_{mc} = \frac{l_m}{\mu S_m} = \frac{1}{G_m}$ and $R_{mB} = \frac{\delta}{S_B} = \frac{1}{G_B}$ are the reluctances of the magnetic path and air gap, respectively.

Thus, we obtain the expression

$$A_e = \frac{(0.4\pi)^2}{8\pi} \int_{G_0}^{G'_B} \frac{(IW)^2}{\left(\frac{G_B}{G_m}+1\right)^2} dG_B \text{ erg.}$$

Assuming IW = const, and as a first approximation $\mu \approx$ const and $G_m \approx$ const, we obtain

$$A_e = \frac{(0.4\pi)^2}{8\pi}(IW)^2 \frac{G_m^2(G'_B - G_0)}{(G'_B + G_m)(G_0 + G_m)} = \frac{(0.4\pi)^2}{8\pi} P \frac{Qf_0}{\rho l_{av}} G_0 \frac{\frac{G'_B}{G_0} - 1}{\left(\frac{G'_B}{G_m}+1\right)\left(\frac{G_0}{G_m}+1\right)} \tag{6}$$

or

$$A_e = 0.05 \cdot 10^8 P\tau_0 \left(\frac{G'_B}{G_0} - 1\right) \frac{1}{\frac{G'_B}{G_m}+1} \text{ erg.} \tag{7}$$

In the last expression

$$\tau_0 = \frac{0.4\pi 10^{-8} W^2}{(R_{mc} + R_{mB0}) R},$$

where

$$R = \frac{\rho l_{av} W^2}{Qf_0}, \quad R_{mc} + R_{mB0} = \frac{1}{G_m} + \frac{1}{G_0} = \frac{\frac{G_0}{G_m}+1}{G_0}. \tag{8}$$

Thus, for a given change in the air gap permeance $G_B$, i.e., for given $G_0 = \frac{S_B}{\delta_0} = \frac{1}{R_{mB0}}$ and $G'_B = \frac{S_B}{\delta'}$ and hence for a given armature displacement $\delta_0 - \delta'$ the required working efficiency of the magnetic system depends on the product $P\tau_0$. If the available power P is small it becomes necessary to have a large $\tau_0$ (i.e., large size); conversely with a small $\tau_0$ (small size) it is necessary to increase P.

Let us now show the relation between the working efficiency and dimensions of a magnetic system. Taking into consideration relationships

$$Q = L_0\left(\frac{D_{ex} - D_0}{2}\right), \quad l_{av} = \pi\left(\frac{D_{ex} + D_0}{2}\right), \quad G_0 = \frac{S_B}{\delta_0},$$
$$G'_B = \frac{S_B}{\delta'}, \quad G_m = \frac{\mu S_m}{l_m}, \quad S_B = \frac{\pi D_B^2}{4}, \quad \frac{D_0}{D_{ex}} = k_0, \quad \frac{D_p}{D_{ex}} = k_B, \tag{9}$$

where $D_0$ is the core diameter, $D_{ex}$ the external diameter of the winding, $D_p$ the diameter of the pole piece and $L_0$ the length of the (coil) winding, we obtain

$$A_e = \left[0.005\pi \frac{f_0}{\rho} \frac{1-k_0}{1+k_0} k_B^2 \frac{1}{\delta_0} \frac{\frac{\delta_0}{\delta'} - 1}{\left(\frac{l_m S_B}{\delta' \mu S_m}+1\right)\left(\frac{l_m S_B}{\delta_0 \mu S_m}+1\right)}\right] P L_0 D_{ex}^2, \tag{10}$$

or

$$A_e = cPL_0 D_{ex}^2, \tag{11}$$

where c is a coefficient which includes design parameters and is equal to the expression in square brackets in (10).

The working efficiency of the magnetic system can be expressed in still another way. Taking into consideration that $P = \frac{k}{0.24} S_{cool}(\theta_x^\circ - \theta_0^\circ) = k'S_{cool}(\theta_x^\circ - \theta_0^\circ)$, we obtain

$$A_e = 0.02\pi(\theta_x^\circ - \theta_0^\circ) G_\theta G_e G_0 \frac{\frac{\delta_0}{\delta'} - 1}{\left(\frac{l_m S_B}{\delta' \mu S_m} + 1\right)\left(\frac{l_m S_B}{\delta_0 \mu S_m} + 1\right)}, \tag{12}$$

or with $\mu \to \infty$

$$A_e = 0.02\pi(\theta_x^\circ - \theta_0^\circ)\left(\frac{\delta_0}{\delta'} - 1\right) G_\theta G_e G_0. \tag{13}$$

Thus, in this expression its basic parameter, the working efficiency, which characterizes the properties of an electromagnetic system is also expressed in terms of the three conductivities: the magnetic, electrical and thermal.

It is essential in decreasing the size for a given P or $A_e$ to allow for a greater heating of the winding, i.e., to increase $\theta_x^\circ - \theta_0^\circ$.

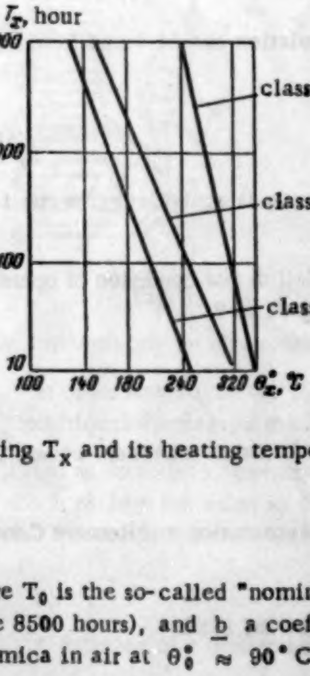
Abroad [3] there appeared recently electromagnetic devices working at temperatures from − 65 up to 250 to 500° C. The following winding materials were used:

a) nickel-plated copper wire, ceramic compound ("keroc") covered, with a single or double external teflon covering (this conductor is used up to $\theta_{max}^\circ = 260°$ C);

b) nickel-plated copper wire, double layer fiberglass covered (permissible temperature $\theta_{max}^\circ = 250$ to 260° C);

c) silver wire, double layer fiberglass covered (used up to $\theta_{max}^\circ = 320$ to 350° C);

d) aluminum wire with oxide insulation (used up to $\theta_{max}^\circ \approx 500°$ C when covered with a ceramic compound).




For interlayers tensions of $\Delta U \geq 500$ v spacers between layers are used. For $\theta_{max}^\circ \leq 250°$ C the following materials are used for spacing: 1) small resin plates used as insulation tape, 2) flaky mica (muscovite or phlogopite), 3) laminated asbestos, 4) laminated teflon, 5) silicon saturated glass fiber. At higher temperatures composite mica, phlogopite, or silicon resin are used. Sometimes the whole winding is heated in silicon resin lacquer.

Winding frames should be made of temperature-stable materials such as special plastics or ceramics. Not only the limiting temperatures, but the linear expansion temperature coefficients of the frame, conductors and covering compounds should be correctly chosen.

In raising the permissible winding temperature it is necessary to consider the life of the winding. The relation between the life of the winding $T_x$ and its heating temperature $\theta_x^\circ$ can be expressed in the form

$$T_x = T_0 e^{-b(\theta_x^\circ - \theta_{prm}^\circ)},$$

where $T_0$ is the so-called "nominal life" under "normal" permissible temperature $\theta_{prm}^\circ$ (usually $T_0$ is taken to be 8500 hours), and b a coefficient depending on the material of the winding insulation. For paper, cotton and mica in air at $\theta_0^\circ \approx 90°$ C, $b \approx 0.071/°$ C.

The graph shows curves of $T_x = f(\theta_x^\circ)$ for various types of insulation.

If the winding has to work under different temperature conditions: at $\theta^{\circ}_{x1}$ for time $t_1$, at $\theta^{\circ}_{x2}$ for $t_2$, at $\theta^{\circ}_{x3}$ for $t_3$ etc., the values of T are determined for each time interval

$$T_{x1} = T_0 e^{-b(\theta^{\circ}_{x1} - \theta^{\circ}_{\text{prm}})},$$

$$T_{x2} = T_0 e^{-b(\theta^{\circ}_{x2} - \theta^{\circ}_{\text{prm}})},$$

$$\cdots\cdots\cdots$$

$$T_{xn} = T_0 e^{-b(\theta^{\circ}_{xn} - \theta^{\circ}_{\text{prm}})},$$

as well as the values

$$x_1 = \frac{t_1}{t_1 + t_2 + \ldots + t_n} = \frac{t_1}{\sum_1^n t_k},$$

$$x_2 = \frac{t_2}{t_1 + t_2 + \ldots + t_n} = \frac{t_2}{\sum_1^n t_k},$$

$$\cdots\cdots\cdots$$

which give the relative value (share) of time for each condition of operation.

The service life of the insulation conforms to the expression

$$T_x = T_{\text{srv}} = \frac{1}{\frac{x_1}{T_{x1}} + \frac{x_2}{T_{x2}} + \ldots + \frac{x_n}{T_{xn}}} = \frac{1}{\sum_1^n \left(\frac{x_k}{T_{xk}}\right)}.$$

It should be noted that if the winding is subjected to mechanical vibrations and shocks the insulation deteriorates quicker. This fact requires the correction of the life of the winding by a certain coefficient equal to $T'_{\text{srv}}/T_{\text{srv}} = k_3$ where $T'_{\text{srv}}$ is the service life of the winding subjected to vibration and shocks.

Hence

$$T'_{\text{srv}} = k_3 T_0 e^{-b(\theta^{\circ}_x - \theta^{\circ}_{\text{prm}})}. \tag{14}$$

If $k_3$ is represented in the form $k_3 = e^{-b\Delta\theta_m}$, the service life of the insulation can be found from the formula

$$T'_{\text{srv}} / T_0 e^{-b\left[\left(\theta^{\circ}_x + \Delta\theta^{\circ}_m\right) - \theta^{\circ}_{\text{prm}}\right]}. \tag{15}$$

The latter expression shows that exposure to mechanical vibrations and shocks is equivalent to a rise in the heating temperature of $\Delta\theta^{\circ}_m$.

The value of $k_3$ must be determined experimentally for each type of insulation and condition of operation. According to existing data it can be considered to lie within the limits $k_3 = 1$ to 3.

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Received October 28, 1957

# SYNTHESIS OF STEP-BY-STEP SELECTOR CIRCUITS

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**The paper describes the construction and principles of operation of step-by-step selectors, and shows that their operation can be expressed algebraically by means of functions, developed for multipositional elements. A synthesis of several step-by-step selector circuits is given as an example.**

Selectors are devices which sometimes form part of automatic circuits. A selector consists of the following elements (Fig. 1):

a) an electromagnet coil, functioning as an ordinary relay through a normally closed contact $\underline{p}$ provided for the purpose;

b) a mechanical device which moves the wipers step-by-step;

c) a selector bank.

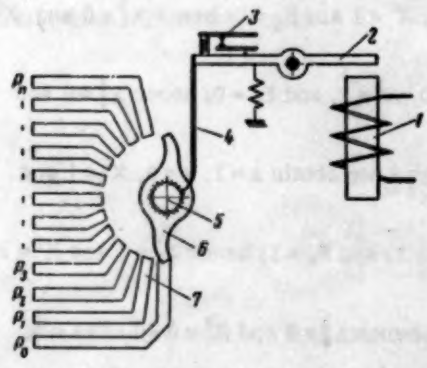


Fig. 1

The operation of the selector is as follows. When a current is set flowing through the electromagnet 1, armature 2 is attracted. In its movement the armature opens contact 3 and simultaneously pulls pawl 4, which falls into the next trough of the ratchet wheel, 5. The system remains in this position as long as there is a current flowing through the winding. When the current stops flowing armature 2 returns to its normal position closing contact 3 and advancing wiper 6 by one step. It follows that with every operation and release of the armature the wipers connect new contacts in 7 in sequential order.

The process of armature attraction is known as the operation of the selector.

Knowing the operation of the selector it is possible to write immediately its characteristic equation which will be expressed as a function of its own contact $\underline{p}$.

In order to analyze the operation of the selector let us note that owing to its wipers it operates as a multipositional element in the circuit.

Let us establish correspondence between a selector with n + 1 positions and n + 1 variables $X^0, \ldots, X^n$ [1] which assume the value of 0 or 1 in such a way that $X^i = 1$ if the i-th contact is closed and in all other cases $X^i = 0$. Moreover, the variables $X^i$ are connected by the relationships for multipositional elements [2-5]:

$$\Sigma X^i = 1, \quad X^i X^j = 0 \quad (i \neq j).$$

A function of a multipositional element is written in the form [5]

$$f = f(0)\,X^0 + \ldots + f(n)\,X^n.$$

Let $\underline{x}$ be the selector's own contact. The characteristic equation of the selector ([1], p. 202) has the form

$$X_N^i = (x_N + \bar{x}_{N-1})\,X_{N-1}^i + \bar{x}_N x_{N-1} X_{N-1}^{i-1}.$$

It expresses the fact that each release after operation [6] ($x_N x_{N-1} = 1$) moves the wipers one step forward.

Let us demonstrate on a few examples how to synthesize selector circuits.

Fig. 2

Example I. Let us construct a circuit with one selector X and one pushbutton A operating as follows.

The selector advances one step for each operation of pushbutton A. It is assumed that the selector has two positions and that $X^0$ and $X^1$ are its bank contacts.

The function of the selector's own contact operation has the form:

$$F_x = ax\,(\lambda_0^1 X^0 + \lambda_1^1 X^1) + a\bar{x}\,(\lambda_0^2 X^0 + \lambda_1^2 X^1) + \bar{a}x\,(\lambda_0^3 X^0 + \lambda_1^3 X^1) + \bar{a}\,\bar{x}\,(\lambda_0^4 X^0 + \lambda_1^4 X^1),$$

where $\lambda_j^i$ is the unknown which should be determined.

The characteristic equation of the selector has the form

$$X_N^k = (x_N + \bar{x}_{N-1})\,X_{N-1}^k + \bar{x}_N x_{N-1} X_{N-1}^{k-1}.$$

At rest $a = 0$, $x = 0$, $X^0 = 1$, $F_X = 0$; hence $\lambda_0^4 = 0$ and $X^1 = 0 + 1 \cdot 0 \cdot 1 = 0$.

From this position as the result of the operation of pushbutton A we obtain $a = 1$, $x = 0$, $X^0 = 1$ and $F_X = 1$; hence $\lambda_0^2 = 1$ and $X^1 = 0 + 0 \cdot 0 \cdot 1 = 0$.

In this position (place) the selector remains operated $a = 1$, $x = 1$, $X^0 = 1$ and $F_X = 1$; hence $\lambda_0^1 = 1$, $X^1 = 0 + 0 \cdot 1 \cdot 1 = 0$.

From this position by releasing button A we obtain $a = 0$, $x = 1$, $X^0 = 1$ and $F_X = 0$; hence $\lambda_0^3 = 0$ and $X^1 = 0 + 1 \cdot 1 \cdot 1 = 1$.

In this position the selector is in a stable condition, $a = 0$, $x = 0$, $X^1 = 1$, and $F_X = 0$; hence $\lambda_1^4 = 0$ and $X^0 = 0 + 1 \cdot 0 \cdot 1 = 0$.

From this position as the result of a new operation of pushbutton A we obtain $a = 1$, $x = 0$, $X^1 = 1$ and $F_X = 1$; hence $\lambda_1^2 = 1$ and $X^0 = 0 + 0 \cdot 0 \cdot 1 = 0$.

In this position the selector remains operated, $a = 1$, $x = 1$, $X^1 = 1$, and $F_X = 1$; hence $\lambda_1^1 = 1$ and $X^0 = 0 + 0 \cdot 1 \cdot 1 = 0$.

Releasing pushbutton A we have $a = 0$, $x = 1$, $X^1 = 1$ and $F_X = 0$; hence $\lambda_1^3 = 0$ and $X^0 = 0 + 1 \cdot 1 \cdot 1 = 1$.

In this position the selector is in a stable condition, $a = 0$, $x = 0$, $X^0 = 1$ and $F_X = 0$; hence $\lambda_0^4 = 0$ and $X^1 = 0 + 1 \cdot 0 \cdot 1 = 0$.

With a new operation the cycle repeats itself. Hence

$$F_x = ax\,(X^0 + X^1) + a\bar{x}\,(X^0 + X^1).$$

But $X^0 + X^1 = 1$, hence $F_X = ax + a\bar{x} = a$. The circuit is shown in Fig. 2.

Example II. Let us construct a circuit with one selector X and one pushbutton A operating as follows.

When pushbutton A is operated the wiper advances one step; when the pushbutton is released the wiper advances another step. The cycle can be repeated indefinitely. It is assumed that the selector has two positions.

The function of the selector's own contact operation has the form:

$$F_x = ax(\lambda_0^1 X^0 + \lambda_1^1 X^1 + \lambda_2^1 X^2 + \ldots + \lambda_{2n-1}^1 X^{2n-1}) + a\bar{x}(\lambda_0^2 X^0 + \lambda_1^2 X^1 + \lambda_2^2 X^2 + \ldots + \lambda_{2n-1}^2 X^{2n-1}) + \bar{a}x(\lambda_0^3 X^0 + \lambda_1^3 X^1 + \lambda_2^3 X^2 + \ldots + \lambda_{2n-1}^3 X^{2n-1}) + \bar{a}\bar{x}(\lambda_0^4 X^0 + \lambda_1^4 X^1 + \lambda_2^4 X^2 + \ldots + \lambda_{2n-1}^4 X^{2n-1}),$$

where $\lambda_j^i$ is the unknown which is to be found.

The characteristic equation will take the form

$$X_N^h = (x_N + \bar{x}_{N-1}) X_{N-1}^h + \bar{x}_N x_{N-1} X_{N-1}^{h-1}.$$

The given program requires the fulfillment of the following conditions.

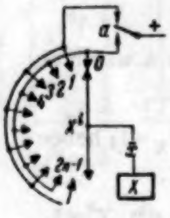


Fig. 3

0. In the state of rest we have $a = 0$, $x = 0$, $X^0 = 1$, $F_X = 0$; hence, $\lambda_0^4 = 0$ and $X^1 = 0 + 1 \cdot 0 \cdot 1 = 0$.

I. From this position operating A we obtain $a = 1$, $x = 0$, $X^0 = 1$, $F_X = 1$; hence $\lambda_0^2 = 1$ and $X^1 = 0 + 0 \cdot 0 \cdot 1 = 0$.

II. From this position the selector wipers advance one step, $a = 1$, $x = 1$, $X^0 = 1$, $F_X = 0$; hence $\lambda_0^1 = 0$ and $X^1 = 0 + 1 \cdot 1 \cdot 1 = 1$.

III. In this position the selector remains in a stable condition, $a = 1$, $x = 0$, $X^1 = 1$, $F_X = 0$; hence $\lambda_1^2 = 0$, and $X^2 = 0 + 1 \cdot 0 \cdot 1 = 0$.

IV. Releasing pushbutton A we have $a = 0$, $x = 0$, $X^1 = 1$, $F_X = 1$; hence $\lambda_1^4 = 0$ and $X^2 = 0 + 0 \cdot 0 \cdot 1 = 0$.

V. From this position the selector advances one step, $a = 0$, $x = 1$, $X^1 = 1$, $F_X = 0$; hence $\lambda_1^3 = 0$ and $X^2 = 0 + 1 \cdot 1 \cdot 1 = 1$.

VI. In this position the selector remains in a stable condition, $a = 0$, $x = 0$, $X^2 = 1$, $F_X = 0$; hence $\lambda_2^4 = 0$ and $X^3 = 0 + 1 \cdot 0 \cdot 1 = 0$.

VII. With subsequent operations of pushbutton A the cycle repeats itself until the whole of the bank has been traversed. Thus, we obtain

$$\lambda_0^1 = \lambda_2^1 = \lambda_4^1 = \lambda_6^1 = \ldots = \lambda_{2n-2}^1 = 0,$$
$$\lambda_1^2 = \lambda_3^2 = \lambda_5^2 = \lambda_7^2 = \ldots = \lambda_{2n-1}^2 = 0,$$
$$\lambda_0^2 = \lambda_2^2 = \lambda_4^2 = \lambda_6^2 = \ldots = \lambda_{2n-2}^2 = 1,$$
$$\lambda_1^3 = \lambda_3^3 = \lambda_5^3 = \lambda_7^3 = \ldots = \lambda_{2n-1}^3 = 0,$$
$$\lambda_1^4 = \lambda_3^4 = \lambda_5^4 = \lambda_7^4 = \ldots = \lambda_{2n-1}^4 = 1,$$
$$\lambda_0^4 = \lambda_2^4 = \lambda_4^4 = \lambda_6^4 = \ldots = \lambda_{2n-2}^4 = 0.$$

The recurring equation is in this case written as follows:

$$F_x = ax \sum_1^n \lambda_{2i-1}^1 X^{2i-1} + a\bar{x} \sum_1^n X^{2(i-1)} + \bar{a}x \sum_1^n \lambda_{2(i-1)}^3 X^{2(i-1)} + \bar{a}\bar{x} \sum_1^n X^{2i-1}.$$

One of the solutions for $\lambda_{2i-1}^1 = \lambda_{2(i-1)}^3 = 0$ is $F_x = \bar{x}\left[a \sum_1^n X^{2(i-1)} + \bar{a} \sum_1^n X^{2i-1}\right]$, which gives the circuit shown in Fig. 3.

Example III. Let us construct a circuit with one selector X and one pushbutton A operating as follows.

When pushbutton A is operated the selector is energized; when pushbutton A is released the selector wiper advances one step. With a new operation of pushbutton A the selector is again energized. When the pushbutton is released the wipers run over the remaining bank contacts in their proper order and stop in the initial position; the selector has n positions.

The function of the selector operation has the form:

$$F_x = ax\sum_0^n \lambda_i^1 X^i + a\bar{x}\sum_0^n \lambda_i^2 X^i + \bar{a}x\sum_0^n \lambda_i^3 X^i + \bar{a}\bar{x}\sum_0^n \lambda_i^4 X^i,$$

where $\lambda_j^i$ are the unknown.

The characteristic equation of selector has the form

$$X_N^k = (x_N + \bar{x}_{N-1})\, x_{N-1}^k + \bar{x}_N x_{N-1} X_{N-1}^{k-1}.$$

The following program must be fulfilled.

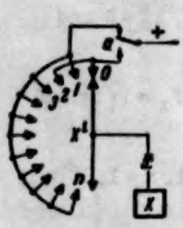

Fig. 4

0. In the state of rest we have $a = 0$, $x = 0$, $X^0 = 1$, $F_x = 0$; hence, $\lambda_0^4 = 0$ and $X^1 = 0 + 1\cdot 0\cdot 1 = 0$.

I. From this position as the result of the operation of pushbutton A we obtain $a = 1$, $x = 0$, $X^1 = 1$, $F_x = 1$; hence, $\lambda_0^2 = 1$ and $X^0 = 0 + 0\cdot 0\cdot 1 = 0$.

II. The selector assumes a stable position, $a = 1$, $x = 1$, $X^0 = 1$, $F_x = 1$; hence $\lambda_0^1 = 1$ and $X^1 = 0 + 0\cdot 1\cdot 1 = 0$.

III. From this position releasing pushbutton A we obtain $a = 0$, $x = 0$, $X^0 = 1$, $F_x = 0$; hence $\lambda_0^3 = 0$ and $X^1 = 0 + 1\cdot 1\cdot 1 = 1$.

IV. The selector assumes a stable position, $a = 0$, $x = 0$, $X^1 = 1$, $F_x = 0$; hence, $\lambda_1^4 = 0$ and $X^2 = 0 + 1\cdot 0\cdot 1 = 0$.

V. From this position by operating pushbutton A the selector is energized, $a = 1$, $x = 0$, $X^1 = 1$, $F_x = 1$; hence, $\lambda_1^2 = 1$ and $X^2 = 0 + 0\cdot 0\cdot 1 = 0$.

VI. In this position the selector assumes a stable condition $a = 1$, $x = 1$, $X^1 = 1$, $F_x = 1$; hence, $\lambda_1^1 = 1$ and $X^2 = 0 + 0\cdot 1\cdot 1 = 0$.

VII. In this position the selector assumes a stable condition, $a = 0$, $x = 1$, $X^1 = 1$, $F_x = 0$; hence $\lambda_1^3 = 0$ and $X^2 = 0 + 1\cdot 1\cdot 1 = 1$.

VIII. From this position releasing pushbutton A the selector is energized again, $a = 0$, $x = 0$, $X^2 = 1$, $F_x = 1$; hence $\lambda_2^4 = 1$ and $X^3 = 0 + 0\cdot 0\cdot 1 = 0$.

IX. From this position the selector wiper advance one step $a = 0$, $x = 1$, $X^2 = 1$, $F_x = 0$; hence $\lambda_2^3 = 0$ and $X^3 = 0 + 1\cdot 1\cdot 1 = 1$.

X. From this position the selector again repeats in their proper order cycles VIII and IX affording one the opportunity to determine the remaining required parameters.

As a result we have

$$\begin{aligned}
&\lambda_0^1 = \lambda_2^1 = 0,\\
&\lambda_0^2 = \lambda_2^2 = 1,\\
&\lambda_1^2 = \lambda_3^2 = 0,\\
&\lambda_0^3 = \lambda_1^3 = \lambda_3^3 = \lambda_4^3 = \ldots = \lambda_n^3 = 0,\\
&\lambda_0^4 = \lambda_2^4 = \lambda_4^4 = \lambda_6^4 = \ldots = 0,\\
&\lambda_1^4 = \lambda_3^4 = \lambda_5^4 = \lambda_7^4 = \ldots = 1.
\end{aligned}$$

The function of the selector operation has the form:

$$F_x = ax\sum \lambda_{2i-1}^1 X^{2i-1} + a\bar{x}\left(X^0 + X^2 + \sum_4^n \lambda_i^2 X^i\right) +$$

$$+ \bar{a}x\lambda_2^3 X^2 + \bar{a}\bar{x}\left(X^1 + X^3 + \sum_4^n X^i\right).$$

Assuming the undetermined parameters to be equal to zero we obtain a single solution

$$F_x = x\left[a\,(X^0 + X^2) + \bar{a}\left(X^1 + X^3 + \sum_4^n X^i\right)\right],$$

which is given in the circuit shown in Fig. 4.

Example IV. Let us construct a circuit, with two five position selectors $\underline{x}$ and $\underline{y}$, which behaves as follows when pushbutton A is operated.

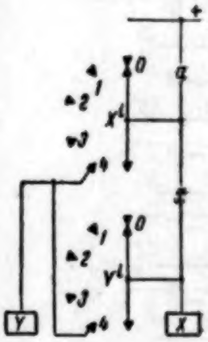


Fig. 5

1. Selector $\underline{x}$ advances five steps and then returns to its initial position. At that instant the wiper of selector $\underline{y}$ advances one step.

2. Selector $\underline{x}$ repeats the cycle, its wipers again advance five steps and return to the initial position. At that instant the wipers of selector $\underline{y}$ advance another step.

3. This operation continues until the wipers of selector $\underline{y}$ close the last contact, when the wipers of selector $\underline{x}$ advance again through the five steps.

The circuit stops when the wipers of both selectors close the last contact.

4. When pushbutton A is released both selectors return to their initial position. The function of the selector operation has the form:

$$F_x = axy\sum\lambda^1_{ij}X^iY^j + ax\bar{y}\sum\lambda^2_{ij}X^iY^j + a\bar{x}y\sum\lambda^3_{ij}X^iY^j + a\bar{x}\bar{y}\sum\lambda^4_{ij}X^iY^j +$$
$$+ \bar{a}xy\sum\lambda^5_{ij}X^iY^j + \bar{a}x\bar{y}\sum\lambda^6_{ij}X^iY^j + \bar{a}\bar{x}y\sum\lambda^7_{ij}X^iY^j + \bar{a}\bar{x}\bar{y}\sum\lambda^8_{ij}X^iY^j,$$
$$F_y = axy\sum\mu^1_{ij}X^iY^j + ax\bar{y}\sum\mu^2_{ij}X^iY^j + a\bar{x}y\sum\mu^3_{ij}X^iY^j + a\bar{x}\bar{y}\sum\mu^4_{ij}X^iY^j +$$
$$+ \bar{a}xy\sum\mu^5_{ij}X^iY^j + \bar{a}x\bar{y}\sum\mu^6_{ij}X^iY^j + \bar{a}\bar{x}y\sum\mu^7_{ij}X^iY^j + \bar{a}\bar{x}\bar{y}\sum\mu^8_{ij}X^iY^j,$$

and the characteristic equation is

$$X^k_N = (x_N + \bar{x}_{N-1})\,X^k_{N-1} + \bar{x}_N x_{N-1} X^{k-1}_{N-1},$$
$$Y^k = (y_N + \bar{y}_{N-1})\,Y^k_{N-1} + \bar{y}_N y_{N-1} Y^{k-1}_{N-1}.$$

The solution found by the method described above is given in the table on the next page.

From the table it follows that

$$F_x = axy\,(\lambda^1_{00}X^0Y^0 + \lambda^1_{10}X^1Y^0 + \lambda^1_{20}X^2Y^0 + \lambda^1_{30}X^3Y^0 + \lambda^1_{01}X^0Y^1 +$$
$$+ \lambda^1_{11}X^1Y^1 + \lambda^1_{21}X^2Y^1 + \lambda^1_{31}X^3Y^1 + \lambda^1_{02}X^0Y^2 + \lambda^1_{12}X^1Y^2 +$$
$$+ \lambda^1_{22}X^2Y^2 + \lambda^1_{32}X^3Y^2 + \lambda^1_{03}X^0Y^3 + \lambda^1_{13}X^1Y^3 + \lambda^1_{23}X^2Y^3 +$$
$$+ \lambda^1_{33}X^3Y^3 + \lambda^1_{04}X^0Y^4 + \lambda^1_{14}X^1Y^4 + \lambda^1_{24}X^2Y^4 + \lambda^1_{34}X^3Y^4 + X^4Y^4) +$$
$$+ ax\bar{y}\,(\lambda^2_{40}X^4Y^0 + \lambda^2_{41}X^4Y^1 + \lambda^2_{12}X^4Y^2 + \lambda^2_{43}X^4Y^3 + \lambda^2_{44}X^4Y^4) +$$
$$+ a\bar{x}y\left(\sum_{i\neq 0}\lambda^3_{ij}X^iY^j + X^0Y^1 + X^0Y^2 + X^0Y^3 + X^0Y^4\right) +$$
$$+ a\bar{x}\bar{y}\,(X^0Y^0 + X^1Y^0 + X^2Y^0 + X^3Y^0 + X^4Y^0) +$$
$$+ \lambda^4_{01}X^0Y^1 + X^1Y^1 + X^2Y^1 + X^3Y^1 + X^4Y^1 + \lambda^4_{02}X^0Y^2 +$$
$$+ X^1Y^2 + X^2Y^2 + X^3Y^2 + X^4Y^2 + \lambda^4_{03}X^0Y^3 + X^1Y^3 + X^2Y^3 +$$
$$+ X^3Y^3 + X^4Y^3 + \lambda^4_{04}X^0Y^4 + X^1Y^4 + X^2Y^4 + X^3Y^4 + X^4Y^4 +$$
$$+ \bar{a}xy\sum_{(\lambda^5_{44}=0)}\lambda^5_{ij}X^iY^j + \bar{a}x\bar{y}\sum\lambda^6_{ij}X^iY^j + \bar{a}\bar{x}y\sum\lambda^7_{ij}X^iY^j + \bar{a}\bar{x}\bar{y}\sum_{(\lambda^8_{00}=0)}\lambda^8_{ij}X^iY^j.$$

| № | $a$ | $x$ | $y$ | $X^i$ | $Y^i$ | $F_x$ | $F_y$ | $\lambda^s_{ij}$ | $\mu^s_{ij}$ | $X^{i+1}$ | $Y^{i+1}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $X^0=1$ | $Y^0=1$ | 0 | 0 | $\lambda^8_{00}=0$ | $\mu^8_{00}=0$ | $X^1=0$ | $Y^1=0$ |
| 1 | 1 | 0 | 0 | $X^0=1$ | $Y^0=1$ | 1 | 0 | $\lambda^4_{00}=1$ | $\mu^4_{00}=0$ | $X^1=0$ | $Y^1=0$ |
| 2 | 1 | 1 | 0 | $X^0=1$ | $Y^0=1$ | 0 | 0 | $\lambda^2_{00}=0$ | $\mu^2_{00}=0$ | $X^1=1$ | $Y^1=0$ |
| 3 | 1 | 0 | 0 | $X^1=1$ | $Y^0=1$ | 1 | 0 | $\lambda^4_{10}=1$ | $\mu^4_{10}=0$ | $X^2=0$ | $Y^1=0$ |
| 4 | 1 | 1 | 0 | $X^1=1$ | $Y^0=1$ | 0 | 0 | $\lambda^2_{10}=0$ | $\mu^2_{10}=0$ | $X^2=1$ | $Y^1=0$ |
| 5 | 1 | 0 | 0 | $X^2=1$ | $Y^0=1$ | 1 | 0 | $\lambda^4_{20}=1$ | $\mu^4_{20}=0$ | $X^3=0$ | $Y^1=0$ |
| 6 | 1 | 1 | 0 | $X^2=1$ | $Y^0=1$ | 0 | 0 | $\lambda^2_{20}=0$ | $\mu^2_{20}=0$ | $X^3=1$ | $Y^1=0$ |
| 7 | 1 | 0 | 0 | $X^3=1$ | $Y^0=1$ | 1 | 0 | $\lambda^4_{30}=1$ | $\mu^4_{30}=0$ | $X^4=0$ | $Y^1=0$ |
| 8 | 1 | 1 | 0 | $X^3=1$ | $Y^0=1$ | 0 | 0 | $\lambda^2_{30}=0$ | $\mu^2_{30}=0$ | $X^4=1$ | $Y^1=0$ |
| 9 | 1 | 0 | 0 | $X^4=1$ | $Y^0=1$ | 1 | 1 | $\lambda^4_{40}=1$ | $\mu^4_{40}=1$ | $X^0=0$ | $Y^1=0$ |
| 0 | 1 | 1 | 1 | $X^4=1$ | $Y^0=1$ | 0 | 1 | $\lambda^1_{40}=0$ | $\mu^1_{40}=1$ | $X^0=1$ | $Y^1=0$ |
| 11 | 1 | 0 | 1 | $X^0=1$ | $Y^0=1$ | 1 | 0 | $\lambda^3_{01}=1$ | $\mu^3_{01}=0$ | $X^1=0$ | $Y^1=1$ |
| 12 | 1 | 1 | 0 | $X^0=1$ | $Y^1=1$ | 0 | 0 | $\lambda^2_{01}=0$ | $\mu^2_{01}=0$ | $X^1=1$ | $Y^2=0$ |
| 13 | 1 | 0 | 0 | $X^1=1$ | $Y^1=1$ | 1 | 0 | $\lambda^4_{11}=1$ | $\mu^4_{11}=0$ | $X^2=0$ | $Y^2=0$ |
| 14 | 1 | 1 | 0 | $X^1=1$ | $Y^1=1$ | 0 | 0 | $\lambda^2_{11}=0$ | $\mu^2_{11}=0$ | $X^2=1$ | $Y^2=0$ |
| 15 | 1 | 0 | 0 | $X^2=1$ | $Y^1=1$ | 1 | 0 | $\lambda^4_{21}=1$ | $\mu^4_{21}=0$ | $X^3=0$ | $Y^2=0$ |
| 16 | 1 | 1 | 0 | $X^2=1$ | $Y^1=1$ | 0 | 0 | $\lambda^2_{21}=0$ | $\mu^2_{21}=0$ | $X^3=1$ | $Y^2=0$ |
| 17 | 1 | 0 | 0 | $X^3=1$ | $Y^1=1$ | 1 | 0 | $\lambda^4_{31}=1$ | $\mu^4_{31}=0$ | $X^4=0$ | $Y^2=0$ |
| 18 | 1 | 1 | 0 | $X^3=1$ | $Y^1=1$ | 0 | 0 | $\lambda^2_{31}=0$ | $\mu^2_{31}=0$ | $X^4=1$ | $Y^2=0$ |
| 19 | 1 | 0 | 0 | $X^4=1$ | $Y^1=0$ | 1 | 1 | $\lambda^4_{41}=1$ | $\mu^4_{41}=1$ | $X^0=0$ | $Y^2=0$ |
| 20 | 1 | 1 | 1 | $X^4=1$ | $Y^1=1$ | 0 | 1 | $\lambda^1_{41}=0$ | $\mu^1_{41}=1$ | $X^0=1$ | $Y^2=0$ |

| № | $a$ | $x$ | $y$ | $X^i$ | $Y^i$ | $F_x$ | $F_y$ | $\lambda^s_{ij}$ | $\mu^s_{ij}$ | $X^i+1$ | $Y^i+1$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 21 | 1 | 0 | 1 | $X^0=1$ | $Y^1=0$ | 1 | 0 | $\lambda^3_{02}=1$ | $\mu^3_{02}=0$ | $X^1=0$ | $Y^2=1$ |
| 22 | 1 | 1 | 0 | $X^0=1$ | $Y^2=1$ | 0 | 0 | $\lambda^2_{02}=0$ | $\mu^2_{02}=0$ | $X^1=1$ | $Y^3=0$ |
| 23 | 1 | 0 | 0 | $X^1=1$ | $Y^2=1$ | 1 | 0 | $\lambda^4_{12}=1$ | $\mu^4_{12}=0$ | $X^2=0$ | $Y^3=0$ |
| 24 | 1 | 1 | 0 | $X^1=1$ | $Y^2=1$ | 0 | 0 | $\lambda^2_{12}=0$ | $\mu^2_{12}=0$ | $X^2=1$ | $Y^3=0$ |
| 25 | 1 | 0 | 0 | $X^2=1$ | $Y^2=1$ | 1 | 0 | $\lambda^4_{22}=1$ | $\mu^4_{22}=0$ | $X^3=0$ | $Y^3=0$ |
| 26 | 1 | 1 | 0 | $X^2=1$ | $Y^2=1$ | 0 | 0 | $\lambda^2_{22}=0$ | $\mu^2_{22}=0$ | $X^3=1$ | $Y^3=0$ |
| 27 | 1 | 0 | 0 | $X^3=1$ | $Y^2=1$ | 1 | 0 | $\lambda^4_{32}=1$ | $\mu^4_{32}=0$ | $X^4=0$ | $Y^3=0$ |
| 28 | 1 | 1 | 0 | $X^3=1$ | $Y^2=1$ | 0 | 0 | $\lambda^2_{32}=0$ | $\mu^2_{32}=0$ | $X^4=1$ | $Y^3=0$ |
| 29 | 1 | 0 | 0 | $X^4=1$ | $Y^2=1$ | 1 | 1 | $\lambda^4_{42}=1$ | $\mu^4_{42}=1$ | $X^0=0$ | $Y^3=0$ |
| 30 | 1 | 1 | 1 | $X^4=1$ | $Y^2=1$ | 0 | 1 | $\lambda^1_{42}=0$ | $\mu^1_{42}=1$ | $X^0=1$ | $Y^3=0$ |
| 31 | 1 | 0 | 1 | $X^0=1$ | $Y^2=1$ | 1 | 0 | $\lambda^3_{03}=1$ | $\mu^3_{03}=0$ | $X^1=0$ | $Y^3=1$ |
| 32 | 1 | 1 | 0 | $X^0=1$ | $Y^3=1$ | 0 | 0 | $\lambda^2_{03}=0$ | $\mu^2_{03}=0$ | $X^1=1$ | $Y^4=0$ |
| 33 | 1 | 0 | 0 | $X^1=1$ | $Y^3=1$ | 1 | 0 | $\lambda^4_{13}=1$ | $\mu^4_{13}=0$ | $X^2=0$ | $Y^4=0$ |
| 34 | 1 | 1 | 0 | $X^1=1$ | $Y^3=1$ | 0 | 0 | $\lambda^2_{13}=0$ | $\mu^2_{13}=0$ | $X^2=1$ | $Y^4=0$ |
| 35 | 1 | 0 | 0 | $X^2=1$ | $Y^3=1$ | 1 | 0 | $\lambda^4_{23}=1$ | $\mu^4_{23}=0$ | $X^3=0$ | $Y^4=0$ |
| 36 | 1 | 1 | 0 | $X^2=1$ | $Y^3=1$ | 0 | 0 | $\lambda^2_{23}=0$ | $\mu^2_{23}=0$ | $X^3=1$ | $Y^4=0$ |
| 37 | 1 | 0 | 0 | $X^3=1$ | $Y^3=1$ | 1 | 0 | $\lambda^4_{33}=1$ | $\mu^4_{33}=0$ | $X^4=0$ | $Y^4=0$ |
| 38 | 1 | 1 | 0 | $X^3=1$ | $Y^3=1$ | 0 | 0 | $\lambda^2_{33}=0$ | $\mu^2_{33}=0$ | $X^4=1$ | $Y^4=0$ |
| 39 | 1 | 0 | 0 | $X^4=1$ | $Y^3=1$ | 1 | 1 | $\lambda^4_{43}=1$ | $\mu^4_{43}=1$ | $X^0=0$ | $Y^4=0$ |
| 40 | 1 | 1 | 1 | $X^4=1$ | $Y^3=1$ | 0 | 1 | $\lambda^1_{43}=0$ | $\mu^1_{43}=1$ | $X^0=1$ | $Y^4=0$ |
| 41 | 1 | 0 | 1 | $X^0=1$ | $Y^3=1$ | 1 | 0 | $\lambda^3_{04}=1$ | $\mu^3_{04}=0$ | $X^1=0$ | $Y^4=1$ |

| № | $a$ | $x$ | $y$ | $X^i$ | $Y^i$ | $F_x$ | $F_y$ | $\lambda^s_{ij}$ | $\mu^s_{ij}$ | $X^i+1$ | $Y^i+1$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 42 | 1 | 1 | 0 | $X^0=1$ | $Y^4=1$ | 0 | 0 | $\lambda^2_{04}=0$ | $\mu^2_{04}=0$ | $X^1=1$ | $Y^0=0$ |
| 43 | 1 | 0 | 0 | $X^1=1$ | $Y^4=1$ | 1 | 0 | $\lambda^4_{14}=1$ | $\mu^4_{14}=0$ | $X^2=0$ | $Y^0=0$ |
| 44 | 1 | 1 | 0 | $X^1=1$ | $Y^4=1$ | 0 | 0 | $\lambda^2_{14}=0$ | $\mu^2_{14}=0$ | $X^2=1$ | $Y^0=0$ |
| 45 | 1 | 0 | 0 | $X^2=1$ | $Y^4=1$ | 1 | 0 | $\lambda^4_{24}=1$ | $\mu^4_{24}=0$ | $X^3=0$ | $Y^0=0$ |
| 46 | 1 | 1 | 0 | $X^2=1$ | $Y^4=1$ | 0 | 0 | $\lambda^2_{24}=0$ | $\mu^2_{24}=0$ | $X^3=1$ | $Y^0=0$ |
| 47 | 1 | 0 | 0 | $X^3=1$ | $Y^4=1$ | 1 | 0 | $\lambda^4_{34}=1$ | $\mu^4_{34}=0$ | $X^4=0$ | $Y^0=0$ |
| 48 | 1 | 1 | 0 | $X^3=1$ | $Y^4=1$ | 0 | 0 | $\lambda^2_{34}=0$ | $\mu^2_{34}=0$ | $X^4=1$ | $Y^0=0$ |
| 49 | 1 | 0 | 0 | $X^4=1$ | $Y^4=1$ | 1 | 1 | $\lambda^4_{44}=1$ | $\mu^4_{44}=1$ | $X^4=1$ | $Y^0=0$ |
| 50 | 1 | 1 | 1 | $X^4=1$ | $Y^4=1$ | 1 | 1 | $\lambda^1_{44}=0$ | $\mu^1_{44}=1$ | $X^4=1$ | $Y^0=0$ |
| 51 | 0 | 1 | 1 | $X^4=1$ | $Y^4=1$ | 0 | 0 | $\lambda^5_{44}=0$ | $\mu^5_{44}=0$ | $X^0=1$ | $Y^0=1$ |
| 52 | 0 | 0 | 0 | $X^0=1$ | $Y^0=1$ | 0 | 0 | $\lambda^8_{00}=0$ | $\mu^8_{00}=0$ | $X^0=1$ | $Y^1=0$ |

A single solution is obtained when it is assumed that $\lambda^2_{44}=\lambda^3_{1j}=\lambda^4_{0j}=1$ and that the parameters which remain undetermined equal zero.

Taking into consideration that $\sum_0^4 X^i=\sum_0^4 Y^i=1$, we obtain

$$F_x=axyX^4Y^4+ax\bar{y}X^4Y^4+a\bar{x}=a(xX^4Y^4+\bar{x})=a(\bar{x}+X^4Y^4). \tag{22}$$

For $F_y$ we can also write

$$\begin{aligned}F_y= axy\,(&\mu^1_{00}X^0Y^0+\mu^1_{10}X^1Y^0+\mu^1_{20}X^2Y^0+\mu^1_{30}X^3Y^0+X^4Y^0+\\
&+\mu^1_{01}X^0Y^1+\mu^1_{11}X^1Y^1+\mu^1_{21}X^2Y^1+\mu^1_{31}X^3Y^1+X^4Y^1+\\
&+\mu^1_{02}X^0Y^2+\mu^1_{12}X^1Y^2+\mu^1_{22}X^2Y^2+\mu^1_{32}X^3Y^2+X^4Y^2+\\
&+\mu^1_{03}X^0Y^3+\mu^1_{13}X^1Y^3+\mu^1_{23}X^2Y^3+\mu^1_{33}X^3Y^3+X^4Y^3+\\
&+\mu^1_{04}X^0Y^4+\mu^1_{14}X^1Y^4+\mu^1_{24}X^2Y^4+\mu^1_{34}X^3Y^4+X^4Y^4)+\\
&+ax\bar{y}\,(\mu^2_{40}X^4Y^0+\mu^2_{41}X^4Y^1+\mu^2_{42}X^4Y^2+\mu^2_{43}X^4Y^3+\mu^2_{44}X^4Y^4)+\\
&+a\bar{x}y\sum_{(\mu_{0j}=0)}\mu^3_{ij}X^iY^j+a\bar{x}\bar{y}\,(X^4Y^0+X^4Y^1+X^4Y^2+X^4Y^3+X^4Y^4+\end{aligned}$$

$$+ \sum_{1}^{4} \mu_{0j}^{4} X^{0} Y^{j}) + \bar{a}xy \sum \mu_{ij}^{5} X^{i} Y^{j} + \bar{a}x\bar{y} \sum \mu_{ij}^{6} X^{i} Y^{j} +$$

$$+ axy \sum \mu_{ij}^{7} X^{i} Y^{j} + \bar{a}\bar{x}\bar{y} \sum_{(\mu_{00}^{8}=0)} \mu_{ij}^{8} X^{i} Y^{j}.$$

Assuming $\mu_{1j}^{2} = \mu_{4j}^{3} = 1$ and the remaining undetermined parameters to be zero we obtain

$$F_{y} = axX^{4} + a\bar{x}X^{4} = aX^{4}.$$

The corresponding circuit is shown in Fig. 5.

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Received November 5, 1957

# STABILITY OF AN INDEPENDENT VOLTAGE AND FREQUENCY AUTOMATIC CONTROL OF A SINGLE SYNCHRONOUS GENERATOR

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**Stability conditions of an independent voltage and frequency automatic control of a single synchronous generator at any established accuracy for either of the controlled values. The investigation methods are based on the theoretical work of M. V. Meerov [1-4].**

It is common knowledge that the dynamic properties of alternating-current motors with a frequency speed control depend to a considerable extent not only on the type of frequency, but also on that of the applied voltage changes. In order to obtain, therefore, the desired working characteristics the alternating current source must be capable of independent frequency and voltage control. When designing frequency and voltage automatic control systems for a synchronous generator one should take into consideration that these two parameters (as a rule) are mutually related and that their interdependence is considerable.

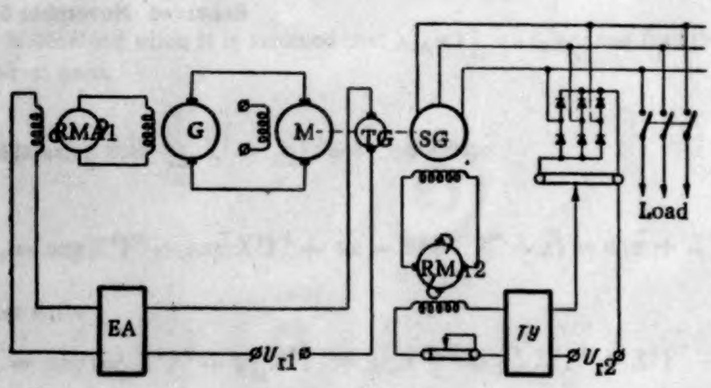


Fig. 1

A dc motor-driven synchronous generator is taken in this paper as the source of the alternating frequency supply. The synchronous generator frequency is controlled by regulating the dc motor speed by means of a tachometer feedback, the first control circuit. In order to obtain high precision of speed control an electronic amplifier is used. The dc motor speed $\underline{n}$ and hence the synchronous generator frequency $f = pn/60$ (where $\underline{p}$ is the number of synchronous generator pole pairs) is set by a reference voltage source $U_{r1}$. For controlling the synchronous generator voltage U, another automatic control system, the second control circuit is used. Reference voltage $U_{r2}$ serves for regulating the automatic voltage control circuit.

A block and schematic diagram for the frequency and voltage automatic control system are shown in Figs. 1 and 2, where RMA is the rotary magnetic amplifier, $k_iT_i$ the first order reactive links, $k_i$ the nonreactive amplifying links, G and M the dc generator and motor, SG the synchronous generator.

Investigations of the stability and independence of the voltage and frequency automatic control systems for the synchronous generator are carried out by means of a linear approximation.

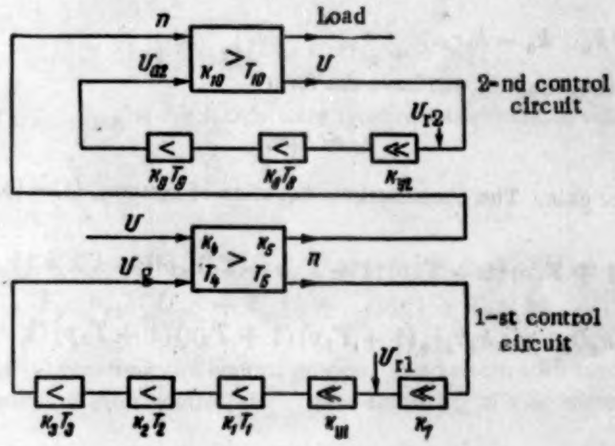


Fig. 2

## Equations for a Frequency and Voltage Automatic Control System of a Synchronous Generator

### 1-st control circuit

Assuming that the rotary magnetic amplifier (RMA1) is completely compensated and taking into account the gain of the electronic amplifier let us write the RMA1 equation in an operator form:

$$(1 + T_1p)(1 + T_2p)U_{a1} = k_1k_2k_y(U_r - e_T), \tag{1}$$

where $T_1$ and $T_2$ are time constants of the control winding and a shorted RMA1 circuit, $k_{g1} = k_1k_2$ the voltage gain of RMA1, $k_y$ the electronic amplifier gain, $U_{a1}$ the output voltage of RMA1 and $e_T$ the tachogenerator voltage.

The equation of the dc generator has the form

$$(1 + T_3p)U_g = k_3U_{a1}, \tag{2}$$

where $T_3$, $U_g$ and $k_3$ are the excitation circuit time constant output voltage and voltage gain of the dc generator, respectively.

The equation of the dc motor, taking the load into consideration, will be

$$(1 + T_5p + T_4T_5p^2)\,n = \frac{U_g}{k_e} - \frac{r_aM_c}{k_ek_m}(1 + T_4p),$$

where $T_4$ and $T_5$ are the dc motor electromagnetic and electromechanical time constants, respectively, $M_c$ the loading torque, $r_a$ the ohmic resistance of the dc motor armature winding, $k_e = C_e\Phi$ and $k_m = C_m\Phi$ characterizing the excitation of the dc motor.

With a linear approximation it is possible to represent the relation between the load $M_c$ and the synchronous generator terminal voltage at a constant wattage by the following expression:

$$M_c = 975\frac{P}{n_0U_0}U = k_cU,$$

where $n_0$ and $U_0$ are the stable values of the motor speed and generator voltage, P the power dissipated in the load, $k_C = 975P/n_0U_0$. Hence the dc motor equation, taking the load into account, will become

$$(1+T_5p+T_4T_5p^2)\,n = k_4U_g - k_4k_5k_6(1+T_4p)\,U, \tag{3}$$

where $k_4 = 1/k_c,\ k_5 = 1/k_M,\ k_6 = k_cr_a$.

The equation of the tachogenerator will have the form

$$e_T = k_7n, \tag{4}$$

where $k_7$ is the tachogenerator gain. The characteristic equation of System (1) - (4) has the form

$$\begin{aligned}&[(1+T_1p)(1+T_2p)(1+T_3p)(1+T_5p+T_4T_5p^2)+k_1k_2k_{y1}k_3k_4k_7]\,n = \\ &= k_1k_2k_{y1}k_3k_4U_{s1} - k_4k_5k_6(1+T_1p)(1+T_2p)(1+T_3p)(1+T_4p)\,U.\end{aligned} \tag{5}$$

2-nd control circuit

Assuming RMA 2 to be completely compensated and taking into account the thyratron amplifier gain, the equation for RMA 2 will have the form:

$$(1+T_8p)(1+T_9p)\,U_{a2} = k_8k_9k_{y2}(U_{s2} - U), \tag{6}$$

where $T_8$ and $T_9$ are the time constants of the control winding and the shorted RMA 2 circuit, $k_8k_9 = k_{g2}$ is the voltage gain of RMA 2, $k_{y2}$ the thyratron amplifier gain, $U_{a2}$ the RMA 2 output voltage.

The equation of the synchronous generator, neglecting transients in the stator circuit, will be

$$(E_d + T_{d0}pE'_d) = E_{de},$$

where $E_d$ is the no-load transient emf, $E'_d$ the emf across the transient reactance, $E_{de}$ the no-load steady state emf, $T_{do}$ the no-load time constant of the synchronous generator excitation winding.

According to the vector diagram of the two reactions of a synchronous generator, it is possible at any moment after connecting the load to assume the following relationship [5]

$$U = \frac{\sqrt{(x_{qc}^2+r_c^2)(x_H^2+r_H^2)}}{x'_{dc}x_{qc}+r_c^2}\,E'_d,\quad E_d = E'_d\,\frac{(x_{dc}x_{qc}+r_c^2)}{x'_{dc}x_{qc}+r_c^2},$$

where $x_{dc} = x_d + x_H,\ x'_{dc} = x'_d + x_H,\ x_{qc} = x_q + x_H,\ r_c = r_a + r_H,\ x_d,\ x'_d,\ x_q$ are reactances and $r_a$ the active resistance of the synchronous generator, $x_H$ and $r_H$ are load parameters. Taking into account these relationships the equation of the synchronous generator can be reduced to the following form:

$$(1+T_{10}p)\,U = k_{10}U_{a2}n, \tag{7}$$

where

$$T_{10} = T_{d0}\frac{x'_{dc}x_{qc}+r_c^2}{x_{dc}x_{qc}+r_c^2},\quad k_{10} = \frac{k_g}{n_0},$$

$$k_g = k_B\frac{\sqrt{(x_{qc}^2+r_c^2)(x_H^2+r_H^2)}}{x_{dc}x_{qc}+r_c^2},$$

$k_B$ is the no-load synchronous generator voltage gain at a nominal speed $n_0$.

Equation (7) contains in its right-hand side a product of variables. Let us represent (7) as a linear approximation by the sum of the first terms of Taylor's expansion. As a result we obtain

$$(1 + T_{10}p) U = k'_{10} U_{a2} + k''_{10} n, \tag{8}$$

where $k'_{10} = k_{10} n_0$, $k''_{10} = k_{10} U_{a\,20}$, $U_{a\,20}$ is the steady state synchronous generator voltage at the excitation winding terminals.

It follows from Equation (6) and (8) that

$$\begin{aligned} &[(1 + T_8 p)(1 + T_9 p)(1 + T_{10} p) + k_8 k_9 k_{y2} k'_{10}] U = \\ &= k_8 k_9 k_{y2} k'_{10} U_{r2} + k''_{10} (1 + T_8 p)(1 + T_9 p) n. \end{aligned} \tag{9}$$

Thus, Equations (5) and (9) represent the control process in a system with two controlled quantities, related to each other by the object of their control, the system consisting of a dc motor and a synchronous generator.

In each of these equations the second right-hand side term represents the coupling operators between the controlled quantities <u>n</u> and U.

## <u>Stability of a Frequency and Voltage Automatic Control System of a Synchronous Generator</u>

Let us clarify how a rise in the gain of one of the circuits and then in both simultaneously influences the stability of the automatic control system under investigation.

On the basis of Equations (5) and (9) the characteristic equation of the system can be written as follows:

$$\begin{aligned} &[(1 + T_1 p)(1 + T_2 p)(1 + T_3 p)(1 + T_5 p + T_4 T_5 p^2)(1 + T_8 p)(1 + T_9 p)(1 + T_{10} p) + \\ &+ k_4 k_5 k_6 k''_{10} (1 + T_1 p)(1 + T_2 p)(1 + T_3 p)(1 + T_4 p)(1 + T_8 p)(1 + T_9 p)] + \\ &+ k_{\text{oxb}\,2} k'_{10} (1 + T_1 p)(1 + T_2 p)(1 + T_3 p)(1 + T_5 p + T_4 T_5 p^2) + \\ &+ k_{\text{oxb}\,1} k_3 k_4 k_7 (1 + T_6 p)(1 + T_9 p)(1 + T_{10} p) + \\ &+ k_{\text{oxb}\,1} k_{\text{oxb}\,2} k_3 k_4 k_7 k'_{10} = 0, \end{aligned} \tag{10}$$

where $k_{\text{oxb}\,1} = k_1 k_2 k_{y1}$, $k_{\text{oxb}\,2} = k_8 k_9 k_{y2}$.

Let us assume that there exists the following coupling between the gains of separate control circuits:

$$k_{\text{oxb}\,2} = \eta k_{\text{oxb}\,1}, \tag{11}$$

where $\eta$ is a constant.

After substituting expression $k_{OXB2}$ from (11) in Equation (10) dividing both parts of it by $k^2_{OXB1}$ and denoting $1/k_{OXB1} = m$, we obtain

$$\begin{aligned} &m^2 [(1 + T_1 p)(1 + T_2 p)(1 + T_3 p)(1 + T_5 p + T_4 T_5 p^2)(1 + T_8 p)(1 + T_9 p) \times \\ &\times (1 + T_{10} p) + k_4 k_5 k_6 k''_{10} (1 + T_1 p)(1 + T_2 p)(1 + T_3 p)(1 + T_4 p)(1 + T_8 p) \times \\ &\times (1 + T_9 p)] + m [\eta k'_{10} (1 + T_1 p)(1 + T_2 p)(1 + T_3 p)(1 + T_5 p + T_4 T_5 p^2) + \\ &+ k_3 k_4 k_7 (1 + T_6 p)(1 + T_9 p)(1 + T_{10} p)] + \eta k_3 k_4 k_7 k'_{10} = 0. \end{aligned}$$

It is obvious that if $k_{OXB1} \rightarrow \infty$, $m \rightarrow 0$.

It has been shown in papers [1-3] that if each minor parameter $\underline{m}$ exceeds the order of the equation by three or more the characteristic equation of the system will always have roots to the right of the imaginary axis and the automatic control system will be unstable. It will be seen from Equation (12) that this is precisely the case; hence with $k_{OXB1} \to \infty$ the automatic control system is unstable. Let us examine the case when $k_{OXB1} \to \infty$, but $k_{OXB2}$ is constant. Equation (10) can then be written as follows

$$m\{[(1+T_1p)(1+T_2p)(1+T_3p)(1+T_5p+T_4T_5p^2)(1+T_8p)(1+T_9p)\times$$
$$\times(1+T_{10}p)+k_4k_5k_6k'_{10}(1+T_1p)(1+T_2p)(1+T_3p)(1+T_4p)(1+T_8p)\times$$
$$\times(1+T_9p)]+k_{\text{oxB}\,2}k'_{10}(1+T_1p)(1+T_2p)(1+T_3p)(1+T_5p+T_4T_5p^2)+$$
$$+k_3k_4k_7(1+T_8p)(1+T_9p)(1+T_{10}p)+k_{\text{oxB}\,2}k_3k_4k_7k'_{10}\}=0. \qquad (13)$$

In this case minor parameter $\underline{m}$ exceeds the order of the equation five times. Hence when $k_{OXB1} \to \infty$ the automatic control system will be unstable.

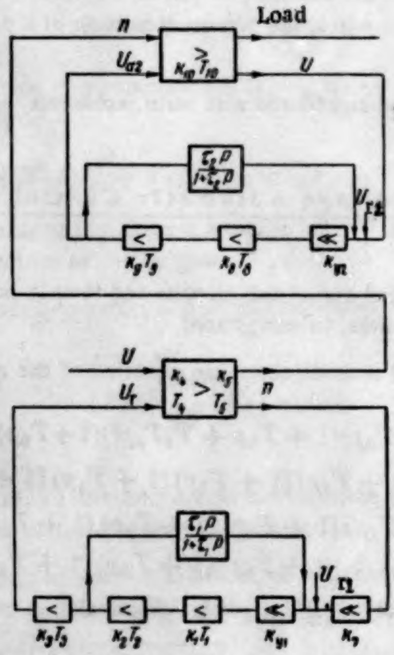


Fig. 3

Let us examine the case when $k_{OXB2} \to \infty$, but $k_{OXB1}$ = const. Denoting $1/k_{OXB2} = m_1$, Equation (10) can be written as follows:

$$m_1\{[(1+T_1p)(1+T_2p)(1+T_3p)(1+T_5p+T_4T_5p^2)(1+T_8p)(1+T_9p)\times$$
$$\times(1+T_{10}p)+k_4k_5k_6k'_{10}(1+T_1p)(1+T_2p)(1+T_3p)(1+T_4p)(1+T_8p)\times$$
$$\times(1+T_9p)]+k_3k_4k_7k_{\text{oxB}\,1}(1+T_8p)(1+T_9p)(1+T_{10}p)+$$
$$+k'_{10}(1+T_1p)(1+T_2p)(1+T_3p)(1+T_5p+T_4T_5p^2)+k_{\text{oxB}\,1}k_3k_4k_7k'_{10}\}=0. \qquad (14)$$

In this case the minor parameter $m_1$ exceeds the order of the equation by three. Hence, when $k_{OXB2} \to \infty$ there will always be at least one root of Equation (14) to the right of the imaginary axis and the system will be unstable.

It follows from the above that with an increase of gain even in one circuit the control of the system as a whole becomes unstable. Hence there arises the problem of finding a control system whose regulation would

remain stable to an accuracy of a minor parameter with an unlimited rise in gain of each of the regulated quantities. It is shown in [3] that if each control circuit represents a structure which will remain stable with an unlimited rise in gain, the whole system with many circuits and interrelated controlled quantities will also remain stable with any gains, if the degenerated and auxiliary equations satisfy the conditions of stability.

On the basis of these considerations let us analyze the stability of a synchronous generator frequency and voltage automatic control system with a flexible feedback incorporating transfer functions of the type $\tau p/(1+\tau p)$, which cover links with the greatest gain. A block schematic of such a system is shown in Fig. 3 and its control process is represented by the following equation:

$$\begin{gathered}
[(1+T_1p)(1+T_2p)(1+T_3p)(1+T_5p+T_4T_5p^2)(1+\tau_1p)+ \\
+k_{\text{охв}\,1}\tau_1p(1+T_3p)(1+T_5p+T_4T_5p^2)+k_{\text{охв}1}\;k_3k_4k_7(1+\tau_1p)]\,n= \\
=k_{\text{охв}\,1}k_3k_4(1+\tau_1p)U_{\text{вт}1}-[k_4k_5k_6(1+T_1p)(1+T_2p)(1+T_3p)(1+T_4p)\times \\
\times(1+\tau_1p)+k_{\text{охв}\,1}k_4k_5k_6\tau_1p(1+T_3p)(1+T_4p)]\,U, \qquad (15) \\
[(1+T_8p)(1+T_9p)(1+T_{10}p)(1+\tau_2p)+k_{\text{охв}\,2}\tau_2p(1+T_{10}p)+ \\
+k_{\text{охв}\,2}k_{11}(1+\tau_2p)]\,U=k_{\text{охв}\,2}k'_{10}(1+\tau_2p)U_{\text{вт}\,2}+ \\
+[(1+T_8p)(1+T_9p)(1+\tau_2p)k'_{10}+k_{\text{охв}\,2}k'_{10}\tau_2p]\,n. \qquad (16)
\end{gathered}$$

Assuming the discrete control circuit gains, whose values can change, are interrelated by the expression

$$k_{\text{охв}\,2}=\eta k_{\text{охв}\,1}, \qquad (17)$$

and introducing notation $1/k_{OXB1}=m$ let us write the characteristic equation of the (15) and (16) system as follows:

$$\begin{gathered}
m^2[(1+T_1p)(1+T_2p)(1+T_3p)(1+T_5p+T_4T_5p^2)(1+T_8p)(1+T_9p)\times \\
\times(1+T_{10}p)(1+\tau_1p)(1+\tau_2p)+k_4k_5k_6k'_{10}(1+T_1p)(1+T_2p)(1+T_3p)\times \\
\times(1+T_4p)(1+T_8p)(1+T_9p)(1+\tau_1p)(1+\tau_2p)]+m\{\eta(1+T_1p)(1+T_2p)\times \\
\times(1+T_3p)[k'_{10}(1+T_5p+T_4T_5p^2)(1+\tau_1p)(1+\tau_2p)+(T_4T_5p^2+T_5p+1)\times \\
\times(1+T_{10}p)(1+\tau_1p)\tau_2p+k_4k_5k_6k'_{10}(1+T_4p)(1+\tau_1p)\tau_2p]+ \\
+(1+T_8p)(1+T_9p)[(1+T_3p)(1+T_5p+T_4T_5p^2)(1+T_{10}p)(1+\tau_2p)\tau_1p+ \\
+k_3k_4k_7(1+T_{10}p)(1+\tau_1p)(1+\tau_2p)+k_4k_5k_6k'_{10}(1+T_3p)(1+T_4p)\times \\
\times(1+\tau_2p)\tau_1p]\}+\eta\{[(1+T_3p)(1+T_5p+T_4T_5p^2)(1+T_{10}p)+ \\
+k_4k_5k_6k'_{10}(1+T_3p)(1+T_4p)]\tau_1p\tau_2p+k_3k_4k_7(1+T_{10}p)(1+\tau_1p)\tau_2p+ \\
+k'_{10}(1+T_3p)(1+T_5p+T_4T_5p^2)(1+\tau_2p)\tau_1p+ \\
+k_3k_4k_7k'_{10}(1+\tau_1p)(1+\tau_2p)\}=0. \qquad (18)
\end{gathered}$$

In Equation (18) each minor parameter $\underline{m}$ raises the order of the equation by two. Hence, according to the conclusions of [3], this system will be stable if the degenerated equation obtained from (18) by making $m=0$ and the auxiliary equation of the second order satisfy the conditions of stability. The technique of compiling an auxiliary equation of the second order is described in [3]. Thus, assuming $k_{OXB1}\to\infty$, which corresponds with $m\to 0$, we shall obtain from (18) a degenerated equation of the form:

$$\begin{gathered}
(1+T_3p)(1+T_5p+T_4T_5p^2)(1+T_{10}p)+k_4k_5k_6k'_{10}(1+T_3p)(1+T_4p)\tau_1p\tau_2p+ \\
+k_3k_4k_7(1+T_{10}p)(1+\tau_1p)\tau_2p+k'_{10}(1+T_3p)(1+T_5p+T_4T_5p^2)(1+\tau_2p)\times \\
\times\tau_1p+k_3k_4k_7k'_{10}(1+\tau_1p)(1+\tau_2p)=0. \qquad (19)
\end{gathered}$$

The auxiliary equation of the second order for (18) will have the form:

$$a_{00}q^5 + m^{1/2}a_{01}q^4 + a_{10}q^3 + m^{1/2}a_{11}q^2 + a_{20}q + m^{1/2}a_{21} = 0, \tag{20}$$

where $q = \sqrt{mp}$,

$$a_{00} = T_1T_2T_3T'_4T'_5T_8T_9T_{10}\tau_1\tau_2,$$

$$\begin{aligned}
a_{01} = {} & T_2T_3T'_4T'_5T_8T_9T_{10}\tau_1\tau_2 + T_1T_3T'_4T'_5T_8T_9T_{10}\tau_1\tau_2 + \\
& + T_1T_2T'_4T'_5T_8T_9T_{10}\tau_1\tau_2 + T_1T_2T_3T'_5T_8T_9T_{10}\tau_1\tau_2 + \\
& + T_1T_2T_3T'_4T_8T_9T_{10}\tau_1\tau_2 + T_1T_2T_3T'_4T'_5T_9T_{10}\tau_1\tau_2 + \\
& + T_1T_2T_3T'_4T'_5T_8T_{10}\tau_1\tau_2 + T_1T_2T_3T'_4T'_5T_8T_9\tau_1\tau_2 + \\
& + T_1T_2T_3T'_4T'_5T_8T_9T_{10}\tau_2 + T_1T_2T_3T'_4T'_5T_8T_9T_{10}\tau_1,
\end{aligned}$$

$$a_{10} = \eta T_1T_2T_3T'_4T'_5T_{10}\tau_1\tau_2 + T_3T'_4T'_5T_8T_9T_{10}\tau_1\tau_2,$$

$$\begin{aligned}
a_{11} = {} & \eta(T_2T_3T'_4T'_5T_{10}\tau_1\tau_2 + T_1T_3T'_4T'_5T_{10}\tau_1\tau_2 + T_1T_2T'_4T'_5T_{10}\tau_1\tau_2 + \\
& + T_1T_2T_3T'_5T_{10}\tau_1\tau_2 + T_1T_2T_3T'_4T_{10}\tau_1\tau_2 + T_1T_2T_3T'_4T'_5\tau_1\tau_2 + \\
& + T_1T_2T_3T'_4T'_5T_{10}\tau_2 + T_1T_2T_3T'_4T'_5T_{10}\tau_1) + \\
& + T'_4T'_5T_8T_9T_{10}\tau_1\tau_2 + T_3T'_5T_8T_9T_{10}\tau_1\tau_2 + T_3T'_4T_8T_9T_{10}\tau_1\tau_2 + \\
& + T_3T'_4T'_5T_9T_{10}\tau_1\tau_2 + T_3T'_4T'_5T_8T_{10}\tau_1\tau_2 + T_3T'_4T'_5T_8T_9\tau_1\tau_2 + \\
& + T_3T'_4T'_5T_8T_9T_{10}\tau_2 + T_3T'_4T'_5T_8T_9T_{10}\tau_1,
\end{aligned}$$

$$a_{20} = \eta T_3T'_4T'_5T_{10}\tau_1\tau_2,$$

$$\begin{aligned}
a_{21} = {} & \eta(T'_4T'_5T_{10}\tau_1\tau_2 + T_3T'_5T_{10}\tau_1\tau_2 + T_3T'_4T_{10}\tau_1\tau_2 + \\
& + T_3T'_4T'_5\tau_1\tau_2 + T_3T'_4T'_5T_{10}\tau_2 + T_3T'_4T'_5T_{10}\tau_1).
\end{aligned}$$

The values of $T'_4$ and $T'_5$ were obtained as the result of expanding

$$(1 + T_5p + T_4T_5p^2) = (1 + T'_4p)(1 + T'_5p).$$

Taking an arbitrary value for the first control circuit stabilizing link time-constant $\tau_1$, we obtain for a chosen structure the second control circuit stabilizing link time-constant $\tau_2$ by means of D-subdivision with respect to parameter $\tau_2$.

By varying the value of $\tau_2$ within the stable region limits of the D-subdivision with respect to $\tau_2$ we attain a stable second order auxiliary Equation (20). Thus, a given automatic control structure with stabilizing links of the type $\tau p/(1 + \tau p)$ with an appropriate choice of parameters $\tau_1$ and $\tau_2$ ensures the required and sufficient conditions for the stability of the automatic control system with 1-st and 2-nd control circuit gains of any arbitrary value of magnitude.

## Independent Frequency and Voltage Control of a Synchronous Generator

Let us examine according to [4] how to obtain an independent frequency and voltage automatic control of a synchronous generator with an accuracy of a minor parameter $m$ by means of increasing the gains of both the control circuits with an additional reaction of extraneous variables on the inputs of the stabilizing links. By extraneous variables we understand the reaction of certain voltage functions on the frequency control circuit and vice versa. The law of auxiliary reactions at the inputs of the stabilizing links will be found from the differential equations which represent the processes in the system.

The schematic and block diagrams of the automatic control system for obtaining an independent voltage and frequency control of a synchronous generator are shown in Figs. 4 and 5.

Let us develop differential equations which would represent the processes of independent frequency and voltage automatic control of a synchronous generator.

The equation for the 1-st control circuit links without a flexible feedback has the form:

$$(1 + T_3p)(1 + T_5p + T_4T_5p^2)\,n = k_3k_4\left[U_{a1} - \frac{k_5k_6}{k_3}(1 + T_3p)(1 + T_4p)\,U\right]. \tag{21}$$

Let us impress on the input of the stabilizing link of the 2-nd control circuit a reaction represented by expression

$$\frac{k_5k_6}{k_3}(1 + T_3p)(1 + T_4p)\,U\,.$$

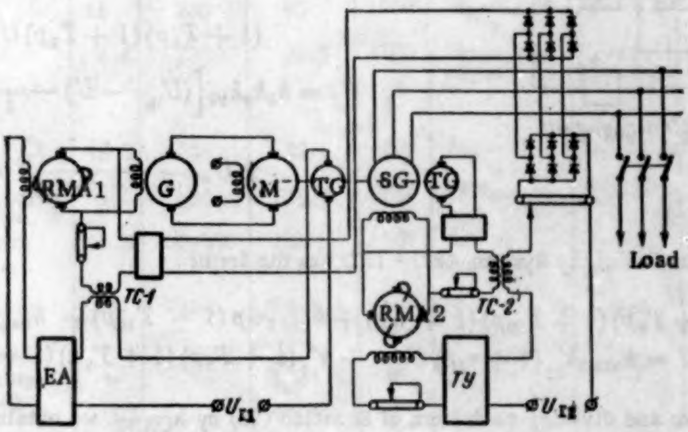


Fig. 4

At the same time the equation of the links subject to flexible feedback will have the form

$$\begin{aligned}(1 + T_1p)(1 + T_2p)\,U_{a1} = k_1k_2k_{y1}\Big[(U_{\text{г}\,1} - e_{\text{т}}) - \frac{\tau_1p}{1 + \tau_1p}U_{a1} + \\ + \frac{\tau_1p}{1 + \tau_1p}\frac{k_5k_6}{k_3}(1 + T_3p)(1 + T_4p)\,U\Big].\end{aligned} \tag{22}$$

The tachogenerator equation will be

$$e_{\text{т}} = k_7n. \tag{23}$$

The characteristic equation of the System (21) - (23) has the form

$$\begin{aligned}&[(1 + T_1p)(1 + T_2p)(1 + T_3p)(1 + T_5p + T_4T_5p^2)(1 + \tau_1p) + \\ &+ k_{\text{охв}\,1}(1 + T_3p)(1 + T_5p + T_4T_5p^2)\tau_1p + k_{\text{охв}\,1}k_3k_4k_7(1 + \tau_1p)]\,n = \\ &= k_{\text{охв}\,1}k_3k_4(1 + \tau_1p)U_{\text{г}1} - k_4k_5k_6(1 + T_1p)(1 + T_2p)(1 + T_3p)\times \\ &\times(1 + T_4p)(1 + \tau_1p)\,U.\end{aligned} \tag{24}$$

Assuming $k_{OXB1} \to \infty$ and dividing every term by $k_{OXB1}$ we obtain the degenerated equation of the following form:

$$\begin{aligned}&[(1 + T_3p)(1 + T_5p + T_4T_5p^2)\tau_1p + k_3k_4k_7(1 + \tau_1p)]\,n = \\ &= k_3k_4U_{\text{г}1}\;(1 + \tau_1p).\end{aligned} \tag{25}$$

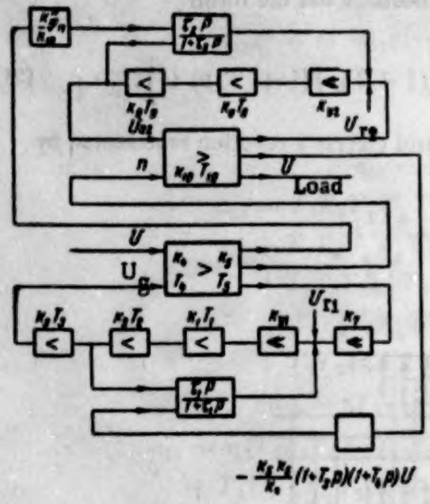


Fig. 5

For the 2-nd control circuit links without feedback the equation will be

$$(1+T_{10}p)\,U = k'_{10}U_{a2} + k''_{10}n = k'_{10}\left(U_{a2} + \frac{k''_{10}}{k'_{10}}\,n\right). \qquad (26)$$

Let us impress on the stabilizing link input a reaction from the 1-st control circuit represented by expression $k_{10}^{n}\, n\, k_{10}^{l}$. At the same time the equation of the links with flexible feedback will be:

$$(1+T_8p)(1+T_9p)\,U_{a2} = k_8k_9k_{y2}\left[(U_{Г_2} - U) - \frac{\tau^2 p}{1+\tau_2 p}\times U_{a2} - \frac{\tau_2 p}{1+\tau_2 p}\,\frac{k''_{10}}{k'_{10}}\,n\right]. \qquad (27)$$

The characteristic equation of the System (26) - (27) has the form:

$$[(1+T_8p)(1+T_9p)(1+T_{10}p)(1+\tau_2p) + k_{oxB2}\tau_2p(1+T_{10}p) + k_{oxB2}k'_{10}\times(1+\tau_2p)]\,U = k_{oxB2}k'_{10}(1+\tau_2p)\,U_{Г_2} + k''_{10}(1+T_8p)(1+T_9p)(1+\tau_2p)\,n. \qquad (28)$$

Assuming $k_{OXB2} \rightarrow \infty$ and dividing each term of Equation (28) by $k_{OXB2}$, we obtain a degenerated equation of the following form:

$$[\tau_2p(1+T_{10}p) + k'_{10}(1+\tau_2p)]\,U = k'_{10}(1+\tau_2p)\,U_{Г_2}\,. \qquad (29)$$

Thus, it will be seen from Equations (25) and (29) that we have obtained in discrete circuits, control processes independent of each other.

## Design and Experimental Section

This section deals with the results of a design and experimental investigation of a frequency and voltage automatic control system of a synchronous generator. The table (page 853) gives technical data of the automatic control system under consideration.

Figure 6 gives a graph obtained from Equation (10) of stability regions against $k_{OXB1}$ (with $k_{OXB2} = 344$) and Fig. 7 that against $k_{OXB2}$ (with $k_{OXB1} = 65$).

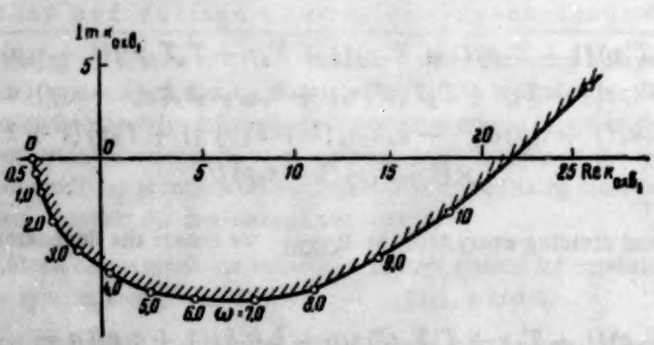


Fig. 6

The critical value of the gain of links covered by the stabilizing links of the 1-st circuit is $k_{OXB1\,crit.} = 22.5$, and for the 2-nd circuit it is $k_{OXB2\,crit.} = 1.93$.

It will be seen that the system is stable at small values of gain in discrete circuits which implies a low accuracy of control.

| List of ACS components | P, kw | U, v | I, a | n, rpm | Gain | Time constants, sec | Remarks |
|---|---|---|---|---|---|---|---|
| Rotating magnetic amplifier No. 1 | 4.5 | 230 | 19.6 | 2890 | $k_1k_2 = 10$ | $T_1 = 0.04$<br>$T_2 = 0.1$ | — |
| dc Generator | 15 | 230 | 65 | 1000 | $k_3 = 3.04$ | $T_3 = 0.3$ | — |
| dc Motor | 4.4 | 220 | 23.5 | 1500 | $k_4 = 7.15$<br>$k_5 = 7.35$ | $T_4 = 0.01$<br>$T_5 = 0.06$ | Dimensions $k_4$, $k_5$ $\frac{rpm}{v}$ |
| Synchronous generator | 7.2 | 230 | 22.6 | 1500 | $k_{10} = 10.3$ | $T_{10} = 0.4$ | |
| Rotating magnetic amplifier No. 2 | 4.5 | 230 | 19.6 | 2850 | $k_8k_9 = 80$ | $T_8 = 0.06$<br>$T_9 = 0.05$ | |
| Induction motor (loading) | 5.3 | 220/380 | 223/13 | 980 | — | — | |
| Stabilizing transformer TS-1 | — | — | 0.3/1.4 | — | — | 0.4 | Transformer type TS 72-60 |
| Stabilizing transformer TS-2 | — | — | 0.3/1.4 | — | — | 0.2 | Transformer type TS 72-60 |
| Tachogenerator No. 1 | — | — | — | — | $k_7 = 0.013$ | — | Dimensions $k_7$ $\frac{v}{rpm}$ |
| Tachogenerator No. 2 | — | — | — | — | $k_T = 0.013$ | — | Dimensions $k_T$ $\frac{v}{rpm}$ |
| Electronic amplifier | — | — | — | — | $k_{y1} = 10$ | — | — |
| Thyratron amplifier | — | — | — | — | $k_{yl} = 4.3$ | — | — |

In order to obtain a highly accurate automatic control system a structure with stabilizing links of the type $\tau p/(1 + \tau p)$ was chosen. With $\tau_1 = 0.4$ sec the D-subdivision with respect to $\tau_2$ was plotted, according to (19) (Fig. 8). It will be seen that the stable region corresponds to any positive real values of $\tau_2$. In compiling a Rowse table for Equation (20), we find the value for parameter $\tau_2 > 0.075$ sec with which the second order auxiliary equation satisfies stability conditions. For our system we have adopted $\tau_2 = 0.2$ sec.

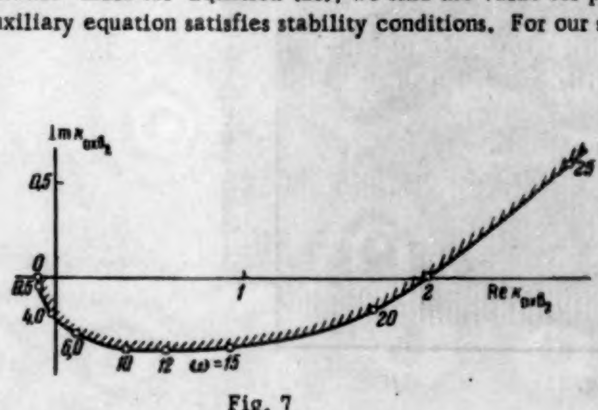


Fig. 7

With the stabilizing device parameters of the 1-st and 2-nd control circuits equalling $\tau_1 = 0.4$ sec and $\tau_2 = 0.2$ sec, respectively, a D-subdivision with respect to $k_{OXB1}$ and $k_{OXB2}$ was plotted (Figs. 9 and 10). It will be seen from the plotted D-subdivisions that stable regions appear at any real values of gain of the 1-st and 2-nd control circuits.

An experimental investigation of the frequency and voltage automatic control system of a synchronous generator was carried out. A disturbance was provided by starting a shorted induction motor of comparable power. Oscillograms (Figs. 11-14) record

**transients in various control circuits. Curve 1 shows the transient in the dc motor speed control circuit, curve 2 shows the external disturbance, and curve 3 the transient in the synchronous generator voltage control circuit.**

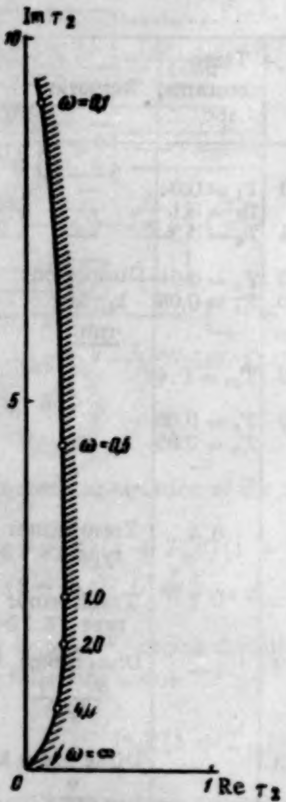


Fig. 8

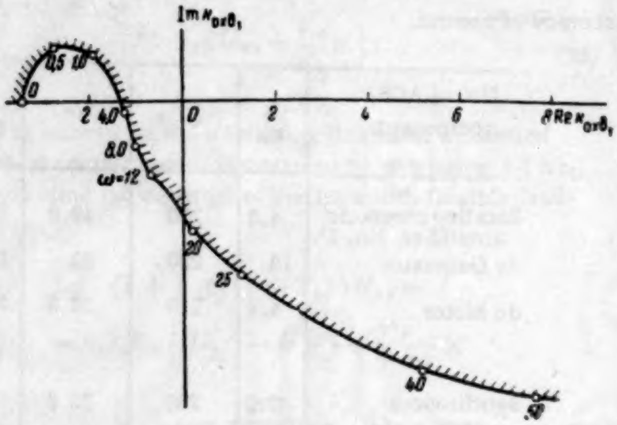


Fig. 9

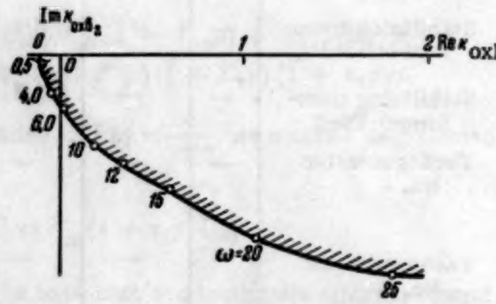


Fig. 10

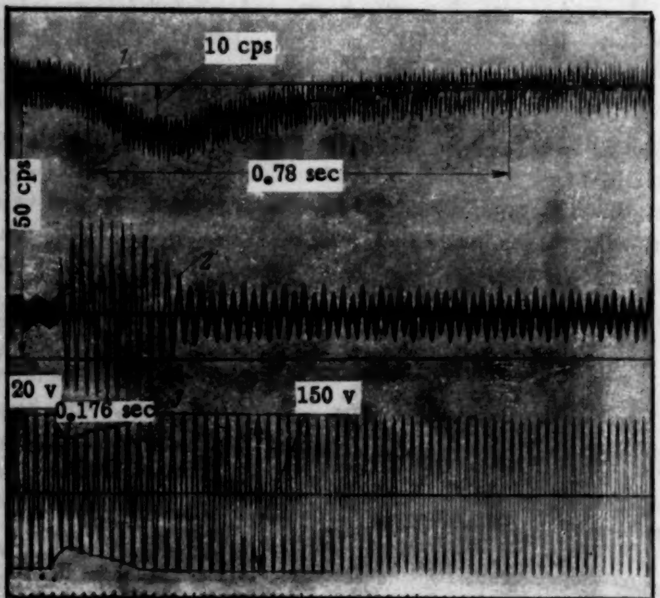


Fig. 11

Oscillograms (Figs. 11, 12, and 13) represent transient processes in both control circuits with a constant gain in the synchronous generator voltage control circuit ($k_g = 3500$) and different gains of the dc motor speed control circuit. The maximum gain of the dc motor armature speed control circuit is $k_m = 280$. This corresponds to the transient shown on the oscillogram of Fig. 13. Oscillograms in Figs. 11, 12, and 13 show a certain independence in the control processes of circuits 1 and 2 due to sufficiently large gains. When the gain of the first control circuit is changed, in the main only the control time in this circuit is changed, whereas the control process in the second circuit is only changed to an insignificant extent.

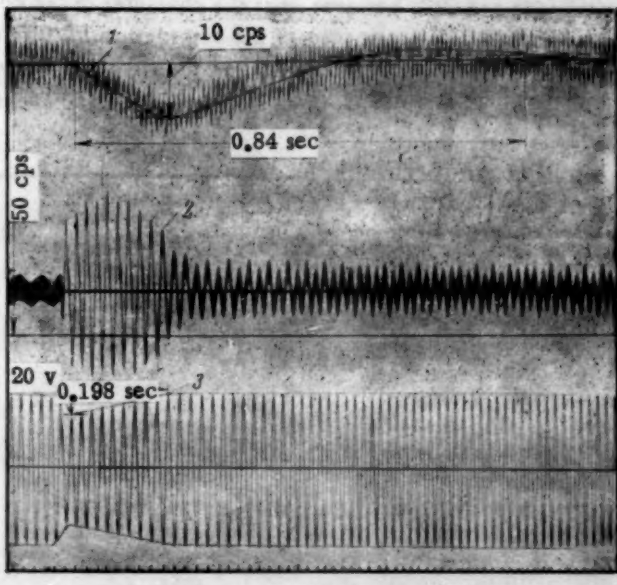


Fig. 12

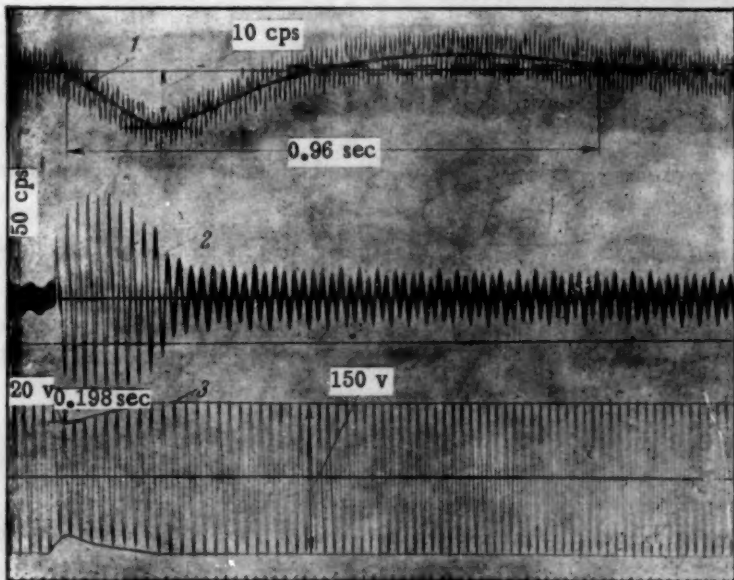


Fig. 13

When additional reaction is impressed on the stabilizing link inputs of the first control circuit according to the law $\frac{k_6 k_8}{k_3}$ $(1 + T_3 p)(1 + T_4 p) U$ and of the second circuit to the law $\frac{k''_{10}}{k'_{10}}$ $n = k_T n$ an independent process of control is ensured to an accuracy of a minor parameter $\underline{m}$ (oscillogram of Fig. 14). If additional reaction is impressed on the inputs of the stabilizing links of the first and second circuits corresponding to the negative and positive signs, the control time in the first circuit increases slightly and in the second circuit it decreases. At the same time the initial deviation of the dc motor armature rotation speed increases as well as the synchronous generator frequency (see oscillogram in Fig. 14). It will be seen that with an independent control of interrelated quantities the control processes of each quantity represented by Equations (25) and (29) are not always the best possible. It follows from Equations (25) and (29), however, that the character of the control processes for each quantity can be influenced by changing certain parameters of the control system. In this case for such parameters (within the limits of a stable operation of the system when choosing the structure) one could use gain $k_7$, which could be changed by means of an additional amplifier, or the time constants $\tau_1$ and $\tau_2$.

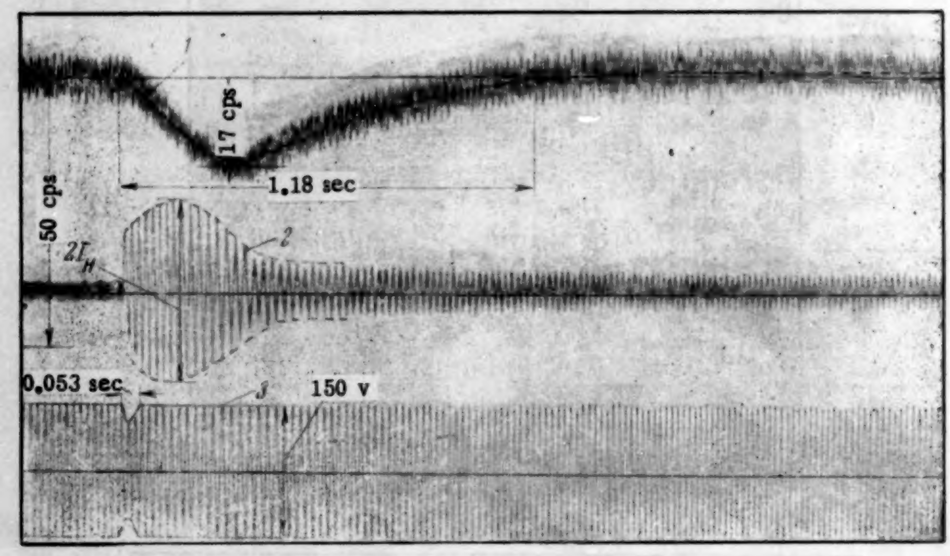


Fig. 14

## SUMMARY


1. An automatic control system structure which ensures high accuracy of frequency and voltage control of a single synchronous generator was chosen.

2. Laws for additional reaction on the stabilizing link inputs of both control circuits with an accuracy of a minor parameter, characterized by the value of the amplification factors, were found.

3. With an independent frequency and voltage control of a single synchronous generator the control processes for each quantity are not always the best possible and depend on the parameters of the degenerated Equations (25) and (29).


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Received October 18, 1957

# CODING AND DECODING DEVICES IN PULSE-CODE TELEMETERING SYSTEMS

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A systematic survey of conversion methods from voltage and current to binary pulse codes and vice versa is given. A comparative evaluation of these methods from the point of view of their applicability to pulse-code telemetering systems is given. Spheres of application of three binary code varieties: the normal, sign-reversing and reflected are shown.

The main problem in designing impulse-code telemetering systems is the development of devices for transforming continuous quantities into discrete ones (coding device) and devices for the reverse operation (decoding devices).

In the technical literature of recent years many similar transformation methods, mainly in electronic digital computers and pulse-code communications systems, are described. In telemetering systems, however, the specific nature of transformations should be taken into account. In digital computers attention is mainly paid to the accuracy of transformations and less to avoiding the circuit and equipment becoming cumbersome.

In communication systems the aim is to obtain very simple and reliable devices, with a sufficient speed of operation; moreover the accuracy of transformation in this case does not play a decisive part.

In telemetering a higher accuracy is required than in communications but lower than that of computers, since the telemetering normal accuracy should be the same as that of local measuring instruments. On the other hand the required speed of operation in telemetering is as a rule lower than that of communications. This is due both to the type of variables measured and the available communication channels (usually a carrier telephone circuit or a narrow-band,high-frequency channel over a power line).

The design of universal pulse-code telemetering systems becomes possible only when all the measured physical quantities are reduced to a single universal electrical parameter.

The present paper is limited to the examination of voltage coding methods and of the reversed transformation (of a pulse-code into voltage).

The process of determining the numerical value of the measured voltage U, called quantizing, consists in determining the number N of units $\Delta U$ which will go into U. The choice of $\Delta U$ determines the accuracy of transformation of voltage U into a discrete signal. The transformation error is determined by the ratio $\delta U = \Delta U/U_{max}$. The coding process consists in forming a combination of pulses corresponding to the obtained number N. In a number of systems the quantizing and coding operations are combined into one.

In principle the application of any code is possible, such as the binary, octonary, decimal, etc. [1]. Moreover the number of pulse attributes in the code must equal the base of the chosen system.

In view of the wide possibilities of using diverse relay operating systems working on the "go- no-go" principle, the binary code is mostly used. The present paper examines the devices for transforming voltage into a binary code. In addition to the normal the sign-reversing and reflected binary codes are also dealt with.

## I. Coding Methods

In the first place let us introduce certain concepts essential for comparing various coding methods.

Methods of code delivery. Two methods of code delivery are possible, the readout and combined methods. In the first method the code is delivered at the completion of the coding process, moreover additional time is required for the readout and sending the code into a channel. In the second method the code is delivered simultaneously with coding. The advantage of the combined method consists in elimination of intervals between coding cycles.

Methods of code transmission to a channel. There exist two methods of transmission the parallel one with frequency [2 and 3] or circuit signal separation, and the consecutive one with time separation of signals. It is obvious that when the code is delivered by the readout method it is possible to transmit pulses either in parallel or consecutively. In the latter case it is possible to change at will the order of sending the code pulses. This constitutes the great advantage of the readout method, since it gives one considerable freedom in the choice of decoding methods.

There exist five basic methods of transforming voltage into a binary code: 1) the pulse counting, 2) "weighing", 3) accumulator discharging, 4) consecutive doubling, and 5) display methods.

1. Pulse counting method.

The essence of this method consists in converting the value of the coded voltage into a number of corresponding pulses (quantizing process), which are recounted by a trigger system into a binary code. Code delivery is done after recounting has been completed. Thus, in pulse-counting coding the operation is done in two periods: the formation of the code and then the readout and transmission.

There exist two varieties of this method: direct and feedback transformation.

Direct transformation. Direct-transforming pulse-counting coding devices count the number of pulses of a given frequency during a determined time interval. Moreover, one of the parameters (frequency or time) is changed proportionately to the coded voltage and the other remains fixed.

A coding system using time-pulse transformation with dynamic compensation (Fig. 1) [4-6] is widely used. The measured voltage $U_x$ is compared by means of a null-indicator HO with a linearly rising voltage of the scanning generator SG. Thus, the voltage is transformed into a proportional time interval. The position of the counting circuit stages at the end of the stable frequency pulse count shows the value of $U_x$ in the binary code. After registering the counting circuit indication and its return to the initial position the cycle repeats itself. $U_x$ can also be transformed into a time interval by means of a phase shifter, whose output voltage phase is displaced with respect to the reference phase by an amount proportional to $U_x$.

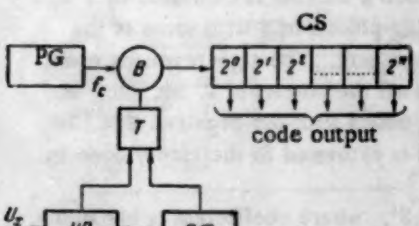



Fig. 1. PG is a pulse generator, B is a gate, CS is a counting circuit, T is a trigger, HO is a null indicator, and SG is a scanning generator.

There also exists a coding device with frequency transformation [7]. As distinct from time transformation circuits, in this instance the counting pulse repetition frequency, used for frequency transformation, is a linear function of the measured quantity, but the time of counting controlled by the timing element is strictly constant.

Transformation with feedback. In coding devices with feedback there are no elements of time or frequency transformation and a stable frequency pulse generator is not required. The feedback contains a decoding device which continuously transforms the binary code numbers registered on the counting circuit into voltage, and the null-indicator compares it with the input voltage. The pulses are counted until the feedback and input voltages become equal.

In the feedback pulse-counting system it is not necessary to return the counting circuit to its zero position after the code is delivered. It is possible to use the final condition of the counting circuit as the initial one for

the next coding cycle. For this purpose a special reversible circuit is used. The speed of operation of such a system increases if $U_x$ changes but little between measurements.

Figure 2 shows a block schematic of a reversible-count feedback coding device. The principal part of the device is the reversible counting circuit where the reversal is obtained by means of electronic gates B, which switch the input of each stage to one of the paraphase outputs of the preceding stage.

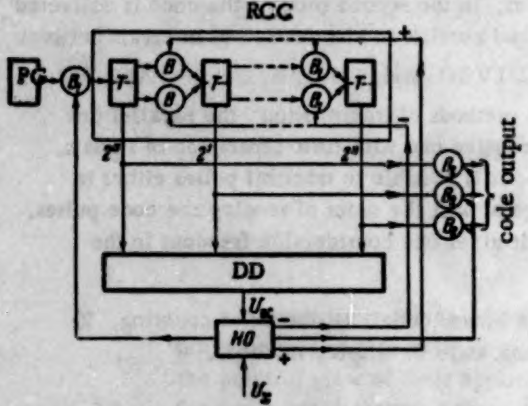



Fig. 2. RCC is a reversible counting circuit, DD is a decoding device, $U_{OC}$ is feedback voltage.

## 2. "Weighing" method.

In this method the coding process is carried out similar to balancing an object on scales by means of weights [4, 8, and 9]. In balancing, weights are selected in a decreasing order of magnitude; similarly in coding voltage $U_x$, it is compared to a sum of reference voltages $2^n\Delta U, 2^{n-1}\Delta U, \ldots, 2^1\Delta U, 2^0\Delta U$, assembled consecutively in the order of the decreasing powers of 2. In a general case the unit criterion in the k-th order of the code (counting from left to right in the decreasing order of indices) is the inequality $U_x \geqslant \sum_{i=n-k+1}^{n} l_i 2^i \Delta U$, where $l_i$ is equal to zero or one, depending on the results of the application of this inequality to preceding terms. The resulting numerical code represents $U_x$ with an accuracy of $\Delta U$. If as a unit criterion of the k-th order of the code we should take the inequality $U_x + \Delta U/2 \geqslant \sum_{i=n-k+1}^{n} l_i 2^i \Delta U$, the error will be $\pm\Delta U/2$. In the above method the quantizing and coding operations are combined.

In the "weighing" method of coding it is often more convenient to use a variety of the binary code, the sign-reversing code. Let us now explain the method of compiling this code.

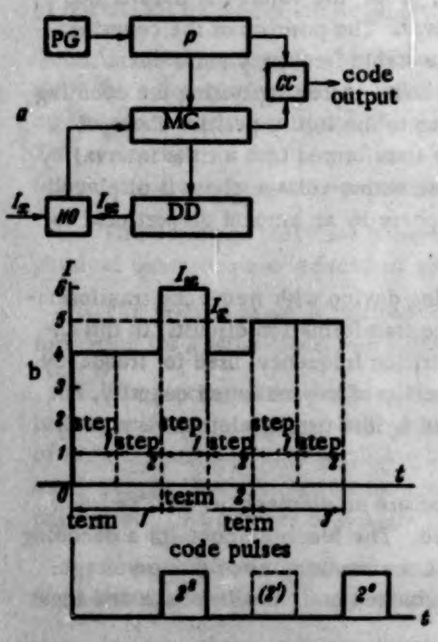



Fig. 3. P is the distributor, CC is the coincidence circuit.

In a normal binary code a number is expressed by a sum of the consecutive decreasing powers of 2 with some of the powers missing from the aggregate. The sign-reversing code, on the other hand, contains all the powers of 2, but some of them with the positive and others with the negative sign. In the general form number N is expressed in the binary code by the summation $N = \sum_{i=0}^{n} l_i 2^i$, where coefficient $l_i$ has in the normal binary code the value of 0 or 1, but in the sign-reversing code that of +1 or − 1. Assuming notation 1 to be a positive and 0 a negative term it is possible, for instance, to obtain an expression for 77 in a sign-reversing code; 77 = 1,100,110. For comparison we shall give the expression for the same number in the normal binary code: 77 = 1,001,101. It should be noted that if the receiving decoding device is of the normal binary type the result of deciphering the received message should be corrected by the formula

$$N = 2N' - (2^{n+1} - 1),$$

where N is the real value of the sent signal, N' the output value of the decoding device, $\underline{n}$ the highest code term (n + 1 the number of terms). Such an operation is easy to realize if the telemetering system and the computer work together.

The advantage of the sign-reversing code as compared with the normal one consists in the greater simplicity of coding by the former in conjunction with the "weighing" method. In order to find out, when compiling a normal binary code, whether the k-th voltage term $2_{n-k+1}\Delta U$ forms part of the reference sum or not, i.e., whether coefficient $l_{n-k+1}$ is one or zero, it is necessary first to add this term to the sum and then to compare the resulting sum with $U_x$. If it is then found that $l_{n-k+1}$ is equal to zero, this term must be subtracted from the sum again. In the sign-reversing code, on the other hand, the sign of the k-th term is determined immediately on obtaining the sum of the preceding (k − 1) terms, i.e.,

$$l_{n-k+1}=\begin{cases} 1 \text{ with } U_x-\sum\limits_{i=n-k+2}^{n} l_i 2^i \dfrac{\Delta U}{2}>0\,, \\ -1 \text{ with } U_x-\sum\limits_{i=n-k+2}^{n} l_i 2^i \dfrac{\Delta U}{2}<0\,. \end{cases}$$

In the case when

$$U_x-\sum_{i=n-k+2}^{n} l_i 2^i \frac{\Delta U}{2}=0\,,$$

the sign of the required k-th term of the reference voltage can be taken arbitrarily since this will only influence the sign of the error (it will be either $+\Delta U/2$ or $-\Delta U/2$).

The block schematic of one of the devices operating according to the "weighing" method is given in Fig. 3, a [10]. In it the reference sum, the feedback current $I_{oc}$, is produced by the decoding device DD which represents the condition of the memory circuit MC stages. The null-indicator HO compares $I_x$ with $I_{oc}$ and according to the sign of their difference sends an instruction for the required stage of the memory circuit MC to remain at 1 or to change over to 0. The time-scale diagram (Fig. 3, b) clarifies the coding process for a three-term binary code of current $I_x$ equal to 5, in nominal units 5 = 101.

### 3. Accumulator discharging method.

In the accumulator discharging method the measured voltage $U_x$ is not permanently connected to the coding device, it is applied to it only periodically for short intervals of time, during which the input reservoir condenser is charged up to $U_x$. During the coding cycle the condenser is discharged to zero by a series of consecutive pulses, which impress on it parts of charges decreasing according to the binary law. Each charge Q is matched by an equivalent change in the voltage across the condenser U = Q/C. Thus, the voltage across the condenser represents the difference between $U_x$ and the reference sum; its sign serves as a criterion for the choice of the next coefficient of the binary code, and hence the next added pulse.

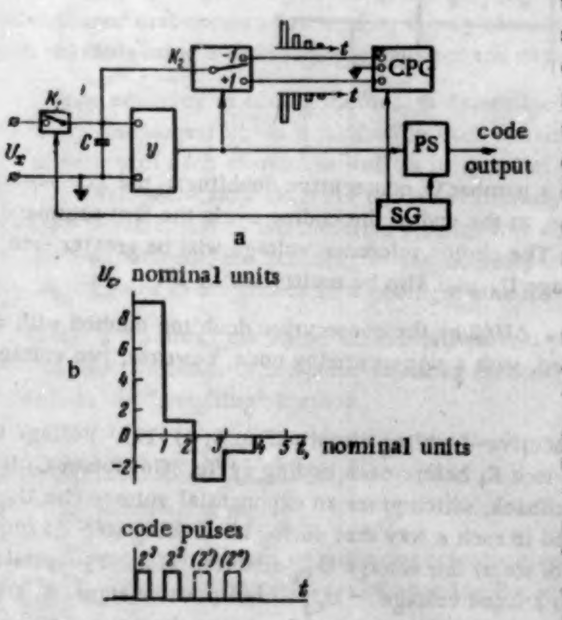



Fig. 4

Figure 4 shows a block schematic of such a device used with a sign-reversing binary code. It comprises a calibrated pulse generator CPG with a paraphase output. The attenuation of pulses according to the binary law is achieved by selecting an appropriate time constant in a special discharge circuit. According to the sign of the voltage on the reservoir condenser C, amplifier Y sends a signal for operating switch $K_2$ into position + 1 or − 1. The condenser will thus receive either a positive or negative calibrated pulse. The pulse shaper PS transmits a code pulse every time the condenser receives a negative calibrated pulse (i.e., when the voltage across the condenser at the previous coding step was positive). The coding process is illustrated on a time scale diagram (Fig. 4, b) by an example of a four-pulse discharge sign-reversing binary code operating with a voltage $U_x$ = 9 and nominal units: 9 = 8 + 4 − 2 − 1.

## 4. Consecutive doubling method [11 and 12].

The essence of this method consists in replacing the consecutive subtraction from $U_x$ of reference voltages, which decrease according to the binary law, by a consecutive subtraction of a constant reference voltage and doubling the difference at each coding step.

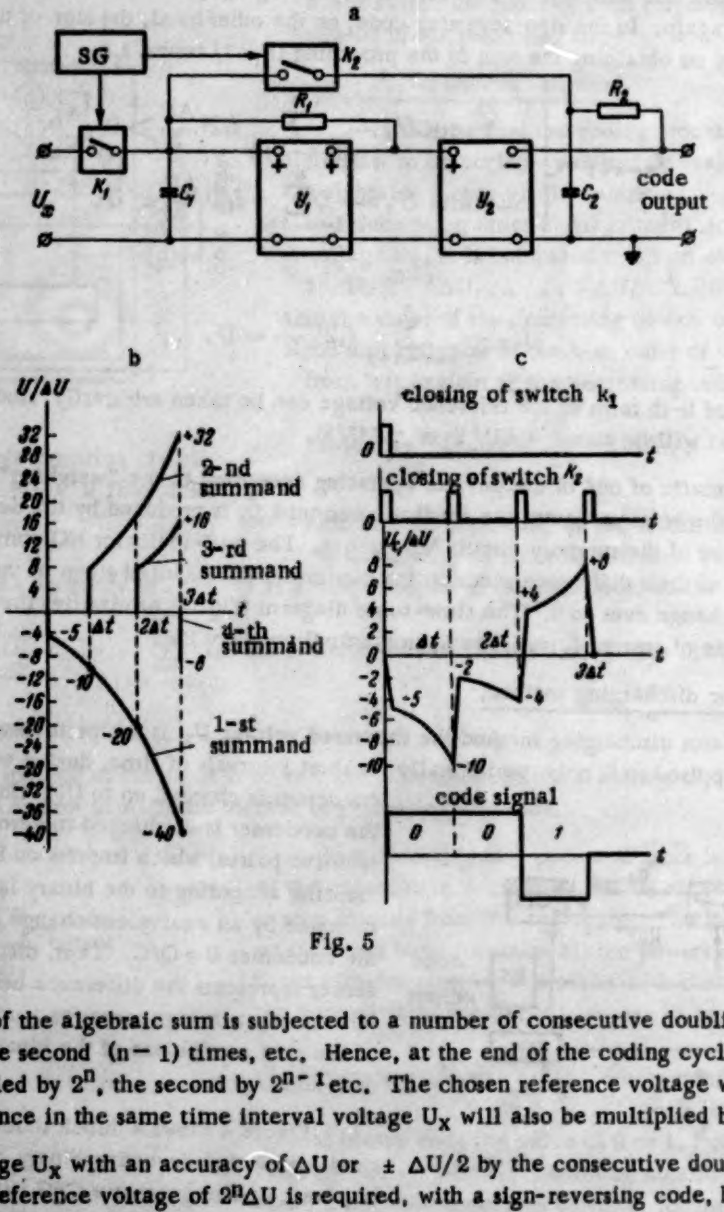



Fig. 5

Each summand of the algebraic sum is subjected to a number of consecutive doublings: the first summand is doubled n times, the second $(n-1)$ times, etc. Hence, at the end of the coding cycle the first summand $2^n \Delta U$ will be multiplied by $2^n$, the second by $2^{n-1}$ etc. The chosen reference voltage will be greater than $\Delta U$ by the factor $2^n$ since in the same time interval voltage $U_x$ will also be multiplied by $2^n$.

For coding voltage $U_x$ with an accuracy of $\Delta U$ or $\pm \Delta U/2$ by the consecutive doubling method with a normal code a single reference voltage of $2^n\Delta U$ is required, with a sign-reversing code, however, two voltages are required $+2^n \Delta U/2$ and $-2^n \Delta U/2$.

Let us now examine one of the sign-reversing consecutive-doubling circuits (Fig. 5, a) [11]. Voltage $U_x$ is impressed on the reservoir condenser $C_1$ by means of switch $K_1$ before each coding cycle. Condenser $C_1$ is connected to the input of amplifier $Y_1$ with a positive feedback, which gives an exponential voltage rise $U_{C_1}$ across the condenser. The values of $R_1$ and $C_1$ are selected in such a way that during one coding step $\Delta t$ (equal to the duration of a coding cycle divided by the number of steps) the voltage $U_{C_1}$ doubles. Block $Y_2$ represents an amplifier with a limiter; it charges condenser $C_2$ up to a fixed voltage $-U_{C_2}$ with a positive signal at the input and to $+U_{C_2}$ with a negative one. At $\Delta t$ time intervals switch $K_2$ is closed for a very short time, transferring the charge from $C_2$ to $C_1$. If condition $C_1 \gg C_2$ is observed the value of the voltage jump on $C_1$ will be very close to $U_{C_2}C_2/C_1$. Voltage $U_{C_2}$ is chosen in such a way as to make $U_{C_2}C_2/C_1 = 2^{n+1} \Delta U$.

Figure 5, a and b shows the process of obtaining a three-pulse charging (n = 2) sign-reversing code for a voltage of $U_x = -5\Delta U$ where b) is the change with time of each summand and c) the variations of the sum $U_{c_1}$. Code pulses without pauses are obtained from amplifier $Y_2$.

5. Display method [4-5].

In the display method, code signals corresponding to discrete values of the $U_X$ level are entered in a table, which is displayed on a cathode-ray tube mask in the form of lines consisting of squares transparent and opaque to the cathode-ray beam. The beam vertical deflection is proportional to $U_X$. Thus, the value of $U_X$ determines the number of the line readout. The electron beam is focused into a horizontal line and displays the squares of a line in parallel. If the table is made up by the normal or the sign-reversing binary code large errors are possible when the electron beam falls onto the boundary of two lines. In this case the delivered code can contain a part of one and a part of the other line. Thus, should the beam fall on the boundary of lines denoting 7 = 0111 and 8 = 1000, code combinations of the type 0001 = 1, 1111 = 15 etc. can arise. A special device automatically displacing the electron beam when it falls on the boundary of two lines complicates the construction. It is more expedient to use Grey's reflected code for the elimination of boundary errors [13].

A display coding mask arranged for a reflected code does not give boundary errors greater than the value of the minor term of the code.

6. Comparative evaluation of coding methods.

In comparing various coding methods it is necessary to take into consideration their following properties: a) the method of code delivery and the possibilities of subsequent decoding, b) accuracy, c) speed of operation.

In the pulse-counting and display coding methods the code is readout. As it has already been pointed out the advantages of this method consist in the greater freedom of choosing decoding devices, and its disadvantage is the necessity to have breaks between coding cycles for sending out the code.

The "weighing," accumulator discharge and consecutive doubling methods allow for a simultaneous method of code delivery. They involve, however, a strictly defined order of code pulses, in the sequence of decreasing "weights."

For evaluating coding methods from the point of view of sending the code to the channel, it should be remembered that consecutive sending is possible with any method; parallel sending, however, is only possible with methods using a discharging condenser and devoid of memory elements.

The accuracy of coding methods is determined by the maximum number of discrete levels (or the number of binary discharges) which is limited in each system by certain factors. The table shows the factors limiting the accuracy of each method as well as its speed of operation. Figures showing the limiting accuracy of each method have been taken from the technical literature dealing with counting technique [5, 6]. It should be noted that the peculiarities of telemetering systems (the absence of highly stable supply sources, impossibility of frequent adjustment, the necessity of utmost compactness, etc.) do not make it always advisable to be guided by these figures in the choice of a coding method.

On the basis of the above considerations it is possible to recommend for use in pulse-code telemetering systems both varieties of the pulse-counting method: the one with direct transformation and with feedback, as well as the "weighing" method.

## II. Decoding Methods

1. Summation method.

The method is based on summing reference voltages or currents "weighted" for the order of the code received. There are several ways of applying the summation method.

Voltage summation. Figure 6 shows a voltage summation decoding circuit based on a computer amplifier with a very high gain and a negative feedback [14]. In it the output voltage is proportional to the sum of the input voltages:

$$U = -\frac{R_1}{R}\sum_{i=0}^{n} l_i 2^i \Delta U, \text{ where } l_i = \begin{cases} 1 & \text{when } k_i \text{ is closed} \\ 0 & \text{when } k_i \text{ is open} \end{cases}$$

Switches $K_0$ to $K_n$ close according to the code received.

| Coding methods | Factors limiting accuracy | Factors limiting speed of operation | Maximum number of binary terms |
|---|---|---|---|
| Impulse-counting method. | | | |
| a) Direct time transformation | Time-pulse converter accuracy and linearity pulse generator stability | Speed of operation of the counting circuit. | 10 |
| b) Direct frequency transformation | Frequency converter accuracy and linearity, timing element accuracy. | ditto | – |
| c) Feedback transformation | Decoding device accuracy, null-indicator sensitivity. | ditto | 8-9 |
| "Weighing" method. | Decoding device accuracy, null-indicator sensitivity. | Distributor speed of operation | 8-9 |
| Accumulator discharge method. | Calibrated pulse generator accuracy, amplifier sensitivity. | Switches speed of operation; time constants of charge-discharge circuits. | 5-6 |
| Consecutive doubling method | Computor amplifier accuracy, time interval stability | ditto | 5-6 |
| Display method | Electron beam thickness. | Counting circuit amplifier bandwidth | – |

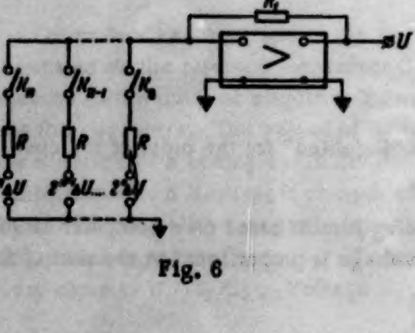


Fig. 6

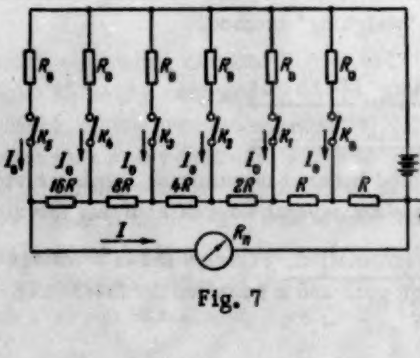


Fig. 7

Current summation. One of the possible current summation circuits is shown in Fig. 7. With the condition $R_0 \gg R \gg R_n$ the total current through the instrument is equal to $I = I_0 \sum_{i=0}^{n} l_i / 2^i$, where $l_i$ is either zero or one.

Charges summation [11]. Figure 8 shows a decoding method based on charging a reservoir condenser similar to the coding method described in part I, section 3. Condenser $C_1$ is charged up to a voltage corresponding to the received code by means of small charges decreasing according to the binary law. The source of these charges, the calibrated pulse generator, sends to the condenser at each decoding step a positive or a negative pulse depending on the value of the given code term. If a normal and not a sign-reversing code is being received pulses of the same sign should be generated and supplied to the condenser only when they coincide with a code pulse, i.e., when a given term is unity. On completion of a decoding cycle a momentary closing of switch $K_2$ transfers the charge from $C_1$ to $C_2$, and then condenser $C_1$ is completely discharged by means of switch $K_3$, thus becoming ready for the next cycle.

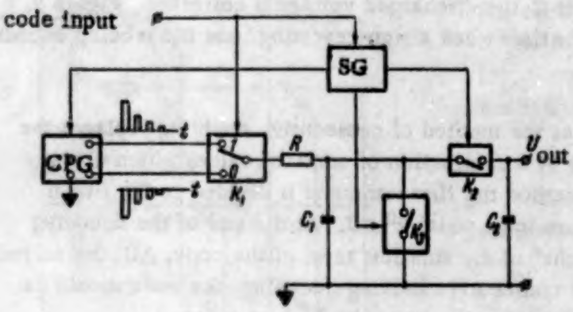
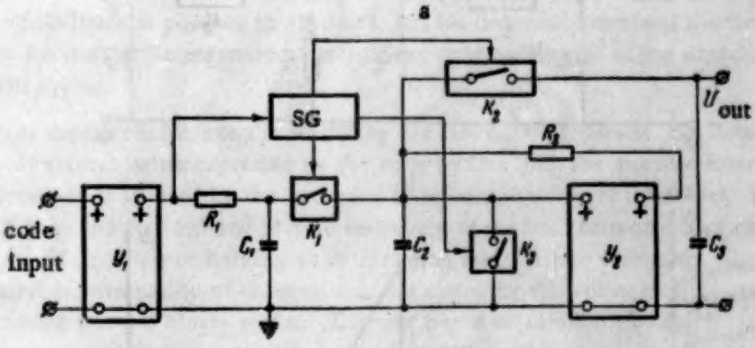



Fig. 8

a




b

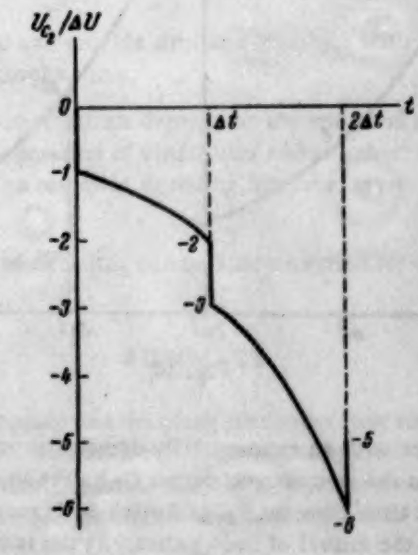



Fig. 9

## 2. Consecutive doubling method.

This decoding method is similar to the coding method described in part I, section 4. Its circuit is shown in Fig. 9, a [11]. In it pulses are summated in condenser $C_2$ and the voltage doubling at each decoding step is achieved by amplifier $Y_2$ which has a positive feedback. Parts of charges constant in value but differing in sign are transferred from condenser $C_2$ to condenser $C_1$, which is connected in the output circuit of the $Y_1$ amplifier with a limiter. Code pulses are impressed on the input of $Y_1$. When the code term is one, $U_{C_1} > 0$ and when it is zero, $U_{C_1} < 0$. Switch $K_1$ is closed momentarily at each decoding step, switches $K_2$ and $K_3$ close one after the other at the end of each decoding cycle. In condenser $C_3$ the discharged voltage is collected. Figure 9, b shows the voltage variations across condenser $C_2$ with respect to time when a sign-reversing code 001 is being decoded.

## 3. Consecutive halving method.

The method of consecutive halving (Fig. 10) as well as the method of consecutive doubling replaces the summation of values calibrated according to the binary law by a summation of constant values which change with time according to the binary law [15]. In the above method the first summand is divided in the first n steps by $2^n$, the second summand by $2^{n-1}$ etc. The total sum must equal $2^n \Delta U$. At the end of the decoding cycle the first summand decreases to the value of the "weight" of the smallest term of the code, $\Delta U$, the second to the "weight" of the second order term, etc. Thus, in the consecutive halving decoding, the code should be arranged in the order of the increasing "weights" of terms, from the lowest to the highest order.

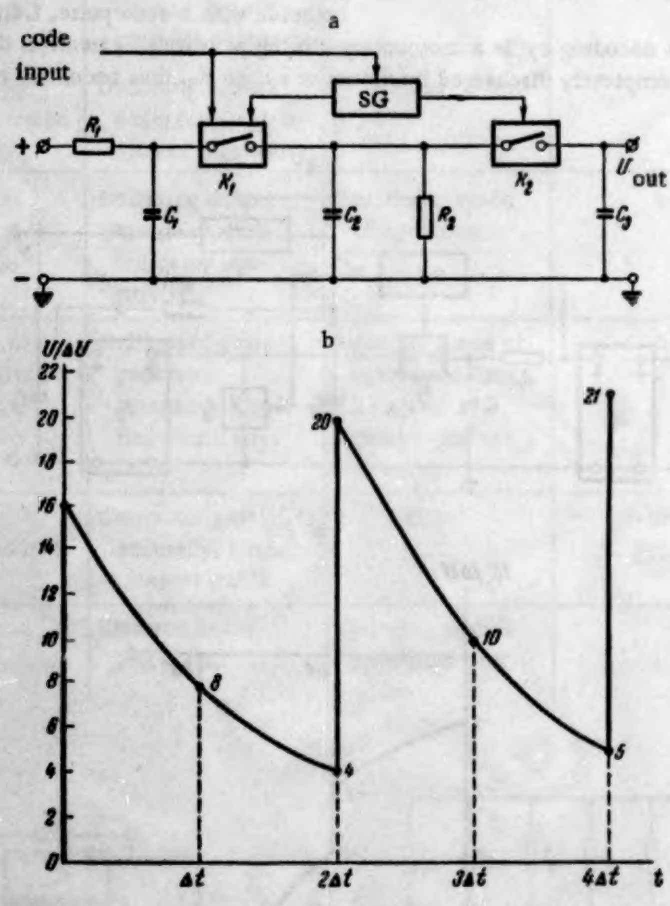



Fig. 10

A circuit of a decoding device with an exponentially decreasing voltage across the reservoir condenser is given in Fig. 10, a. The voltage on the reservoir condenser $C_2$ halves in the time interval $\Delta t$ owing to the specially selected discharge circuit time constant $R_2C_2$. Switch $K_1$ is momentarily closed at the end of the time intervals which coincide with the arrival of code pulses. At the same time voltage $U_{C_2}$ changes instantaneously by the amount $U_1C_1/C_2 = 2^n \Delta U$. Switch $K_2$ is closed at the end of a cycle transferring the result of

decoding to condenser $C_2$. The time scale diagram of changes in $U_{C_2}$ during decoding of a normal binary code 10101 is shown in Fig. 10, b.

4. Matrix method [4].

The matrix method consists in providing a separate output circuit for each code combination. A circuit for a matrix decoding device for a two-pulse discharge code is given in Fig. 11. In it the values of the code terms (0 or 1) are represented by the position of switches $K_1$ and $K_2$. For each code combination one of the horizontal bus bars is connected to the output, the others being shunted to earth. Thus, in the drawing, the top bar is shown connected to the output. The horizontal bus bars receive a series of discrete voltages differing by $\Delta U$.

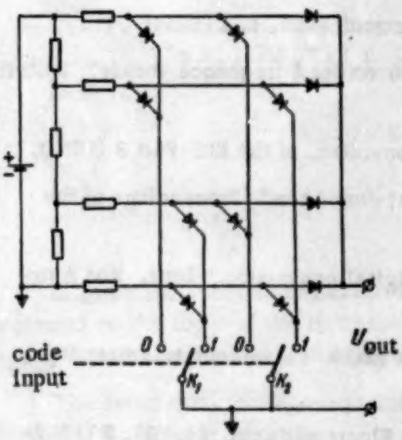



Fig. 11

5. Comparative evaluation of decoding methods.

The same properties are essential for comparing decoding as for coding methods.

With a parallel code transmission all the code terms are received simultaneously in the decoding circuit. This leads to maximum speed. Parallel reception is impossible in devices with consecutive doubling or division methods. With consecutive transmission the terms of the code are received one after the other in the order of decreasing or rising "weights" (in the method of consecutive halving). In this case the output voltage is taken off the decoding device only after completing the decoding cycle. A consecutive application of the code is possible in all cases, but for the summation and the matrix methods memory elements are required for storing the registered terms of the code to the end of the decoding cycle or up to their changing in subsequent cycles.

Decoding devices should conform to the following accuracy requirements: the output voltage (current) must differ from the numerical value expressed by the code by less than the discrete interval value. In the main the decoding accuracy is limited by the accuracy of reference voltages (currents). In the matrix method the accuracy is affected by the forward and reverse resistance of diodes. In methods of consecutive doubling or halving the accuracy of doubling or halving at each coding stage is also essential. The least accurate are decoding methods based on summation of charges, i.e., on changing the voltage of the reservoir condenser. Their accuracy is between 5 and 6 binary terms. Circuits based on summation of voltages or currents have sufficient accuracy for combined operation with any coding device, i.e., they are capable of receiving ten or more terms of the binary code.

Theoretically the matrix method can provide similar accuracy. With a large number of discharges, however, the matrix system becomes cumbersome.

The speed of operation of decoding methods depends on the one hand on the method of supplying the code and on the other on the speed of operation of distributors and switches. It should be noted that for telemetering systems the speed of any of the reviewed decoding methods (as well as the coding methods) is amply sufficient.

The current summation method of decoding can be recommended for use in pulse-code telemetering systems.

## SUMMARY


In the above systematic survey coding and decoding methods which are mainly used in computer and communication techniques are described and their relative evaluation from the point of view of their applicability to pulse-code telemetering systems is given. For coding devices "weighing" and pulse counting methods are recommended, and for decoding the current summation method.

In a number of cases coding devices can be simplified by using a special binary code—the sign-reversing in the case of the "weighing" and accumulator discharging methods, and the reflected in the case of the display

method. It is very important to observe correlation between coding and decoding devices in telemetering systems with respect to their code transmitting and receiving methods. Otherwise additional complications may arise (such as additional memory elements,distributors, etc.).

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Received July 31, 1957.

SELECTIVITY OF RECTIFYING MEASURING DEVICES

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(L'vov)

In many instances when detecting instruments are used in automatic control equipment, the voltage impressed on the input of the instrument contains, together with the signal to be measured, other components such as disturbances of the measuring device itself, or of circuit components preceding the instrument.

The interfering voltages can differ in frequency from the measured signal or be completely aperiodic in character. In order to obtain reliable results the measuring device should not, as far as possible, respond to the interference. Since it is impossible to eliminate interference completely, it becomes important to be able to evaluate the error caused by it.

Henceforth it will be assumed that the measured and interfering voltages are sinusoidal functions of time with frequencies not related harmonically to each other. Formulas for determining errors in normal detecting instruments as well as phase-sensitive rectifiers with different shapes of the reference voltage have also been obtained.

It should be noted that the problem of selectivity of detecting devices has been dealt with in technical literature before [1-4] in connection with investigations of the detecting process in radio receivers.

In this paper we shall quote certain known principles of the theory of detecting devices sensitivity, since these principles have not received due attention in the literature on electrical measurements.

This paper does not deal with the effect of fluctuation noise on measuring instruments, since this is the province of specialized literature [5, 6].

## 1. Selectivity of Nonphase-Sensitive Nonreactive Rectifying Devices

Let the signal voltage impressed on the input of a measuring instrument be $u_x = U_x \sin \omega t$ and that of the interference $u_n = U_n \sin pt$.

The total voltage at the input will be

$$u = U_x + u_i = U_x \sin \omega t + U_i \sin pt. \tag{1}$$

In order to determine the error due to the interference voltage $u_i$ in the simplest rectifying device (Fig. 1) it is necessary to calculate the direct component of the voltage after rectification when voltage $\underline{u}$ is determined by Expression (1).

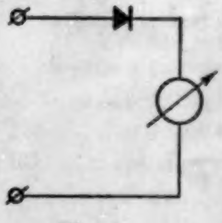

Fig. 1

For determining the direct component it is customary [1] to represent voltage $\underline{u}$ in the form of a voltage of frequency $\omega$, amplitude and phase modulate by a voltage of a difference frequency $\omega - p$. The direct component of the thus-obtained modulated voltage envelope is then taken as the required component. This method as it is pointed out in [1] only holds if the difference frequency is small compared with frequencies $\omega$ and $\underline{p}$, since only in such a case has the concept of an envelope any meaning. In view of the fact, however, that the direct component in the case of detection of two periodic oscillations does not depend on the frequency relationship

[8], but is determined solely by relation $k = U_n/U_x$, the existing formulas for determining the direct component can be applied for calculating the error in question.

Let us denote by $U_0$ the direct component of the voltage impressed on the measuring instrument in the absence of interference.

The value of the voltage with interference will then be determined by expression

$$U = U_0\left[1 + \frac{k^2}{4} + \frac{k^4}{64} + \cdots\right].$$

The case when $k < 1$ is of practical interest. It can be considered then that

$$U = U_0\left(1 + \frac{k^2}{4}\right). \tag{2}$$

The measuring error $\delta$ due to interference will then be determined by the formula

$$\delta = \frac{k^2}{4}.$$

The error of a nonphase-sensitive rectifying device is always positive, whereas (as it will be shown later) that of a phase-sensitive device is negative.

The most important peculiarity of above relationship between $\delta$ and $k$ is the fact that the error grows at a considerably slower rate than the signal-to-noise ratio. In this case the noise is being suppressed by the signal.

With an interference 30% of the measured signal, the error is only 2.3%. Thus, the rectification of the signal before measurement substantially decreases the effect of interference on measurement results. In this respect rectifying instruments have a substantial advantage over those based on the measurement of the effective voltage value such as instruments of the electrodynamic, electromagnetic, thermoelectric and other types.

In fact the reading of an instrument which measures the effective value is proportional to $\sqrt{U_x^2 + U_n^2}$ which is equivalent to $U_x\sqrt{1 + k^2}$. Hence the error due to interference is $\delta = \sqrt{1 + k^2} - 1$ or, for small values of $k$, $\delta = k^2/2$.

Thus, the error due to interference in instruments measuring the effective value is twice as great as that of rectifying instruments.

## 2. Selectivity of Phase-Sensitive Devices With A Sinusoidal Reference Voltage

Let us now determine the error due to interference in phase-sensitive measuring instruments. It is assumed that the reference voltage is also sinusoidal.

A block schematic of any phase-sensitive device can usually be represented as a differential rectifier with one half of it receiving the geometric sum and the other half the difference of the reference and measured voltages (Fig. 2). Moreover if the reference voltage is sufficiently larger than the signal voltage, the direct voltage at the detector output [2] is proportional to $U_x \cos\varphi$, where $\varphi$ is the angle between the reference and the unknown voltages.

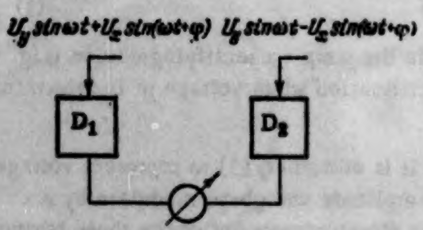

Fig. 2

If the reference voltage amplitude is denoted by $U_y$, the amplitudes of voltages at the inputs of the differential rectifier without interference will be [2]:

$$\begin{aligned} U_c &= \sqrt{U_y^2 + U_x^2 + 2U_0 U_x \cos\varphi}, \\ U_p &= \sqrt{U_y^2 + U_x^2 - 2U_0 U_x \cos\varphi}. \end{aligned} \tag{3}$$

The reading of the instrument at the detector output will be proportional to the difference of amplitudes $U_c - U_p$. The presence of interference with an amplitude $U_i$ will increase the voltage at each half of the detector output. The expression for one half of the amplitude at the input will be [2] $U_c\left(1+\frac{1}{4}\frac{U_1^2}{U_c^2}\right)$, and for the other half $U_p\left(1+\frac{1}{4}\frac{U_1^2}{U_p^2}\right)$.

The instrument reading at the detector output will be proportional to

$$U = U_c\left(1+\frac{1}{4}\frac{U_1^2}{U_c^2}\right) - U_p\left(1+\frac{1}{4}\frac{U_1^2}{U_p^2}\right).$$

Hence the error of a phase-sensitive detector due to interference will be

$$\delta = \frac{U-(U_c-U_p)}{U_c-U_p} = \frac{1}{4}U_1^2\,\frac{\frac{1}{U_c}-\frac{1}{U_p}}{U_c-U_p},$$

whence

$$\delta = -\frac{1}{4}\frac{U_1^2}{U_cU_p}. \qquad (4)$$

The inequality $U_x \ll U_y$ usually holds, hence it is possible to consider with a sufficient degree of accuracy that in Formula (4) $U_cU_p = U_y^2$, i.e.,

$$\delta = -\frac{1}{4}\left(\frac{U_1}{U_y}\right)^2. \qquad (5)$$

It should be immediately noted that the phase-sensitive rectifier error, as distinct from the nonphase-sensitive, is negative. This fact has a simple physical explanation: the interference voltage increases by a larger amount the smaller of the two signal voltages $U_c$ and $U_p$.

Hence interference will decrease the difference between the two voltages which corresponds to the negative sign in Equation (5).

A phase-sensitive instrument error due to interference is only determined by the ratio $U_i/U_y$ and does not depend in the first approximation from the measured voltage $U_x$ or the phase angle.

The form of the Formulas (2) and (4) is similar, with the difference that in (5) ratio $U_i/U_x$ is replaced by $U_i/U_y$. Since normally $U_x = (0.1 \text{ to } 0.2)\ U_y$ the error of a phase-sensitive rectifier, due to the same interference as for a nonphase-sensitive, is some ten times smaller than that of the latter. In order to decrease that error it is obviously advisable to raise the ratio $U_y/U_x$.

## 3. Selectivity of a Phase-Sensitive Device With A Nonsinusoidal Reference Voltage

A comparison of phase-sensitive rectifiers with mechanical ones [7] leads to the idea that the selectivity of phase-sensitive devices could be considerably improved if instead of a sinusoidal reference voltage a rectangular one was used. It is therefore advisable to examine this aspect of phase-sensitive detectors as well.

In practice it is impossible to obtain a completely rectangular voltage shape. Let us assume that the voltage has a trapezoidal shape (Fig. 3). If the reference voltage pulse-time rise is small, i.e., if $\alpha/2\pi \ll 1$ the error due to interference can be calculated on the basis of the previously derived formulas for sinusoidal voltage. For this purpose let us take advantage of the following circumstance: the absolute value of the error due to interference does not depend in a nonphase-sensitive rectifier on the shape of the measured voltage $u_x(t)$ for the time intervals which conform to the inequality:

$$|u_y(t)| > |u_i(t)|. \tag{6}$$

The validity of this statement follows from the fact that for the parts of the period when $|u_y(t)| > |u_i(t)|$ the direct voltage component at the output is either equal to zero with $u_y(t) < 0$, or to the sum of the direct components of $u_y(t)$ and $u_i(t)$. But the direct component of a periodic voltage $u_i(t)$, determined over time intervals which repeat themselves periodically with a frequency not bearing a harmonic relation to the frequency of $u_i(t)$, is equal to zero. Hence the above assertion.

In conclusion we can say that the absolute error due to interference will be the same for signals of trapezoidal or sinusoidal shape, providing the trapezoidal voltage rise and fall curve coincides with the corresponding part of the sine curve. At the same time it is assumed, of course, that the amplitude of the interference voltage $U_i$ is smaller than that of the trapezoidal voltage $U_T$.

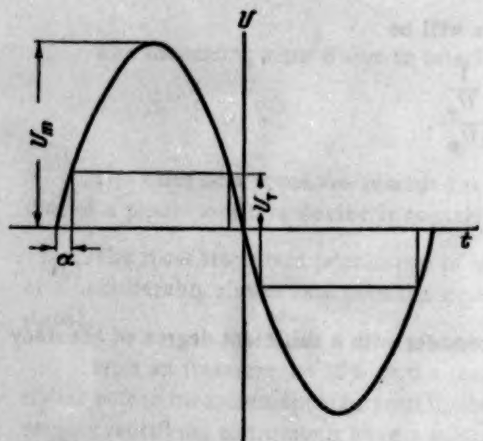


Fig. 3

It is not difficult to calculate the maximum value $U_m$ of the sinusoidal voltage equivalent to the trapezoidal one. From Fig. 3 it becomes obvious that $U_T = U_m \sin\alpha$ and hence

$$U_m = \frac{U_T}{\sin\alpha} \approx \frac{U_T}{\alpha} \tag{7}$$

($\alpha$ is assumed to be a small quantity).

In view of the fact that the absolute value of the phase-sensitive detector output voltage does not depend on the value of the reference voltage, but is determined solely by the value of $U_x \cos\varphi$ the relative errors with trapezoidal and sinusoidal reference voltages are the same in phase-sensitive detectors, providing the voltages conform to relationship (7). We can then obtain from (7) the value of the error of a phase-sensitive detector with a trapezoidal reference voltage in the form:

$$\delta = -\frac{1}{4}\alpha^2 \left(\frac{U_i}{U_T}\right)^2.$$

Since $\alpha$ can be made sufficiently small the error with a trapezoidal reference voltage becomes considerably smaller than with a sinusoidal voltage.

Thus, if $\alpha = 0.1 \approx 6°$ the interference error becomes one hundredth of the one obtained with a sinusoidal reference voltage. In practice this error is so small that with the condition that $U_i < U_y$ and a sufficiently steep reference voltage wave front ($\alpha \leq 0.1$) it can be neglected.

An experimental check completely confirmed above conclusion. With an interference voltage $U_i = (0.8 \text{ to } 0.9)\, U_y$, no noticeable deflection was observed on the output meter of the phase-sensitive instrument, although it registered accurately signal changes of 0.7 to 1 %.

## SUMMARY


1. Rectifying instruments possess the property of suppressing interference by the signal, providing the interference is smaller than the signal.

2. The suppression of interference by the signal is most marked if the detector is a phase-sensitive one.

3. When a phase-sensitive device with a rectangular reference voltage is used the error due to interference becomes negligibly small if the amplitude of the interference is smaller than that of the reference voltage.

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Received June 4, 1957

# CHRONICLE

## ALL-UNION CONFERENCE ON THE THEORY OF RELAY OPERATING DEVICES

The All-Union Conference on the Theory of Relay Operating Devices was held in Moscow from October 3 to 9, 1957. It was dedicated to discussing the possibilities and the results already achieved in this sphere.

The Conference was attended by 335 delegates from 30 towns and 100 organizations, including agencies of the Academy of Sciences of the USSR and Union Republics, Scientific Research Institutes, Schools of Higher Learning, agencies of Economic Councils and various Ministries.

Some foreign specialists took an active part in the work of the Conference, such as Professor Grigore K. Moisil, member of the Academy of the Rumanian People's Republic, Marianna Nedelku and Konstantin Popovich, Scientific Collaborators of the Mathematics Institute of the RPR Academy, engineers George Ioanin and Paul Konstantinesku, Professor Laslo Kal'mar, Corresponding Member of the Academy of the Hungarian People's Republic, and Frantishek Svoboda B. Sc., Scientific Collaborator of the Computer Institute of the Czechoslovak Academy of Sciences.

Thirty-eight papers by representatives of 18 organizations were read and discussed.

Professor A. M. Letov, Deputy Director of the Institute of Automation and Remote Control of the USSR Academy of Sciences, noted in his opening speech that relay operated devices have at present won a permanent place in all the spheres of modern technique. Owing to their simple construction and reliable operation these devices have great possibilities and will play an important role in fulfilling the directives of the XX Congress of the CPSU on the automation of production processes.

Professor M. A. Gavrilov (Institute of Automation and Remote Control of the USSR Academy of Sciences) noted in his paper "Present condition and basic lines of development of the theory of relay circuits" that the present scientific development in the sphere of automatic and remote control and monitoring devices is characterized by the appearance of widely generalized theories. One such theory is that of relay operating devices. The reporter noted that the beginning of the modern theory of relay devices dates to 1938 and that, at present, work on the theory of relay devices is being pursued in almost all the countries of the world.

Having described the present state of the theory of relay operating devices, the author outlined their further development. In his opinion the basic task of the relay devices theory is to examine the relationships prevailing in discrete converters.

Another pressing problem, arising when relay operated devices are synthesized, is that of determining the number and type of elements required for achieving the given operating conditions, the initial structure and the transformation of circuits to the required form.

The wide application of noncontact elements in relay devices requires the creation of a generalized theory of relay circuits.

In conclusion M. A. Gavrilov said a guarantee of the rapid solution of above problems is the wide development of the work on the structural theory of relay devices in many countries of the world.

Professor S. A. Ianovskaia (Moscow State University) surveyed in her paper "Development of mathematical logic and its technical applications" the basic lines of development of mathematical logic and dealt in detail on the reasons for its rapid development, especially noticeable in the last 10 years.

S. A. Ianovskaia showed that the character assumed by the development of mathematical logic, even in the most abstract branch of the foundations of mathematics, makes one reflect on the connection of these abstract problems with the theory of machines and on the fact that mathematical logic itself is of special value for automation.

Academician G. K. Moisil (RPR Academy) noted in his paper "Development of the theory of relay-contact circuits in the Rumanian People's Republic" that the relay-contact algebraic theory was developed in the Rumanian Republic under the direct influence of the Soviet school through the works of B. I. Shestakov and M. A. Gavrilov. In the main Rumanian scientists studied sequential switching circuits by means of a system of recurring equations. The author outlined the basic lines of development pursued in Rumania, the problem of obtaining in synthesized circuits a minimum number of relays for given operating conditions of the final control elements, the study of the operation of various types of relay elements in circuits consisting of these elements, the synthesis of circuits with transients, the problem of simplifying circuits by converting them to bridge operation, etc. In solving the above problems the Galois field theory, the theory of comparisons and multiple logic were used in addition to ambiguous logic. In conclusion Academician G. K. Moisil noted the close collaboration of specialists in the theory of relay circuits with their colleagues in related spheres and those in the Soviet Union.

Several papers dealt with the mathematical aspect of the theory of relay devices.

In his paper "Algebraic theory of the operation of relay-contact circuits" Academician G. K. Moisil showed that the actual operation of circuits sometimes differs considerably from the designed one due to the neglecting of transient processes. The use of special contacts does not always solve the problem. This difficulty can be obviated by means of reversing circuits.

In the paper "Theory of the synthesis of contact circuits" F. Svoboda (Institute of computers of the Czechoslovak Academy of Sciences) showed that the synthesizing process of contact circuits, which satisfies certain obligatory and arbitrary conditions, (conditions of operation, optimum solutions) can be divided into discrete steps which the author named deductive and inductive. The reporter deduced the required conditions for the convergence of the synthetic process. This method was used in constructing the more complex contact circuits in the semi-automatic computer of the Institute of Computers of the Czechoslovak Academy of Sciences.

A paper entitled "Some application of contact grids" was read by F. Svoboda on behalf of A. Svoboda M. Sc. (Institute of computers of the Czechoslovak Academy of Sciences). The paper suggests a method of finding at least one minimum form of a given Boolian function. As the result of transforming this form, other minimum forms are obtained.

Engineer K. Popovich (Mathematic Institute of the Rumanian Academy) dealt in his paper with "A minimum disjunctive form of a Boolian function." The author proposed a more convenient method of representing Boolian functions in a minimum disjunctive form.

On the delegates' request Academician G. K. Moisil read another paper entitled "Synthesis of circuits according to the given conditions of operation." It gave the results of investigation carried out by the author and engineer G. Ioanin with reference to the following problem: how to translate conditions of operation referring to final control elements given in the form of words into a strictly mathematically described program. The proposed method is applicable whenever the worded program states what changes occur to the final control elements each time the receiving elements are affected.

Iu. Ia. Bazilevskii (SKB) read a paper on "Problems of the theory of logical time functions." The author described the basic principles of the mathematical structure of logical time functions, a structure useful for analyzing problems in the sphere of logical designs for machines, automatic and relay devices.

Corresponding Member of the Academy of Sciences, USSR, A. A. Markov (Moscow State University) proved in his report "Inversion complexity of a system of functions" theorems on the meaning of the inversion complexity of Boolian functions and a system of Boolian functions.

S. B. Iablonskii (Applied Mathematics Department of the V. A. Steklov Mathematical Institute of the Academy of Sciences, USSR) read a paper on "Some mathematical problems of the theory of relay circuits" in which he dealt with the basic results of the mathematical investigations in the sphere of the theory of relay circuits, carried out by the mechanico-mathematical faculty of the Moscow State University and the V. A. Steklov Institute of the Academy of Sciences, USSR, in the past year.

T. L. Maistrova (All-Union Correspondence Polytechnical Institute) pointed out in her paper "Application of multiple logic to the theory of relay-contact circuits" the use of the mathematical approach to the analysis and transformation of the P-type circuits which contain relays in addition to contacts.

A. V. Kuznetsov (Electronic analog laboratory of the VINITI of the Academy of Sciences, USSR) proved in his paper "Some problems of the mathematical theory of contact circuits" that it is impossible to construct an algebraic structure with a finite number of basic functions for the whole class of contact circuits which would describe their functional characteristics and connections as adequately as the Boolian algebra structure does it for the P-circuit.

Considerable attention was paid at the conference to methods of synthesizing and analyzing relay circuits of a general type and also of special types.

Professor M. A. Gavrilov in his paper "Design of relay circuits with bridge connections" pointed out that this problem is one of the most important for obtaining a minimum number of relays for a given circuit.

In many cases it leads to a considerable reduction in the number of elements required for the design of a circuit.

The method of designing bridge circuits described in this paper provides for sensitive elements in the middle of the circuit, which theoretically leads to a large saving in equipment.

P. Konstantinesku (Bucharest State University) read a paper on "Synthesis of contact multipoles." The idea of the method consists in the following: on the basis of known admittances between all the terminals, structural P-type formulas are compiled for each pair. Normal methods of synthesis are used for this purpose. By means of these expressions another matrix is made up which contains besides the admittances between terminals of the given program, other admittances between additional terminals introduced by the designer according to certain rules. The author illustrated this method of synthesis by numerous examples.

V. N. Roginskii (Laboratories for the solution of scientific problems in wire communications of the Academy of Sciences, USSR) pointed out in his paper "A graphical method for constructing contact (1, k)-poles" the absence of a sufficiently good engineering method of synthesis of bridge circuits when neutral conditions are taken into account. The method proposed by the author permits one to synthesize contact bridge circuits. This method was worked out by the author in conjunction with G. N. Povarov on the basis of the cascade method.

V. I. Shestakov (Moscow State University) described in his paper "An algebraic method of analyzing and synthesizing multiple relay systems" a method of analyzing and synthesizing circuits with two- and multi-positional elements by means of vector equations.

A. G. Lunts (Leningrad Railway Transport Institute) read a paper on "The matrix method and the method of characteristic functions in the theory of contact circuits." This method permits one to synthesize contact multipoles.

V. N. Grebenshchikov (Moscow State University) proposed in a paper of the same title a method of synthesizing multipole relay-contact circuits by means of known Boolian functions of admittances between its poles. This method is applicable when the given admittances satisfy certain inequalities and it lends itself to the synthesis of nonrecurrent circuits.

Ia. I. Mekler (MOPEO GPI Tiazhpromelektroproekt) in his paper entitled "A graphical method of constructing relay-contact circuits" proposed a method which permits one to obtain direct from the connections table the final or near-final analytical expression for the circuit structure.

A. N. Iurosov (All-Union Correspondence Polytechnical Institute) in a paper entitled "Synthesis of multiple circuits based on the on and off periods" proposed a construction method based on the assumption that each condition (on or off) of a given relay can be in principle ensured by a single operating element during a consecutive number of periods.

V. G. Lazarev (Laboratories of the Academy of Sciences, USSR, for the solution of problems in wire communications) described in his paper "The technique of determining the minimum number of relays required for the construction of relay circuits on the basis of given operating conditions" the results of his investigations in this sphere. The problem of constructing relay-contact circuits with a minimum number of relays is reduced to the problem of determining the minimum number of receiving and intermediate elements.

Many papers dealt with problems of application of the theory of relay circuits to the analysis, synthesis and transformation of circuits with other elements of relay operation such as magnetic, transformer, choke, electronic, rectifier and other devices.

Corresponding Member of the USSR Academy of Sciences, A. A. Markov read a paper entitled "Minimized rectifier-switching circuits representing monotonic symmetrical functions." The author proved the following result he obtained: with any values of $\underline{n}$ and $\underline{m}$ which hold with $0 \leq m \leq n+1$ function $S_m^n$ can be represented by a dipole of above type with $m(n+1-m)$ contacts, and cannot be represented by a dipole of this type with a smaller number of contacts.

M. Nedelku (Mathematical Institute of the Rumanian PR Academy) proposed in a paper entitled "AC-fed electronic relay-operating and rectifier circuits" a method of constructing characteristic equations for analyzing and synthesizing electronic circuits.

V. N. Roginskii discussed in his paper "Equipollent transformations of relay circuits" relay circuits which contain, as distinct from contact circuits, relay winding, and other active resistances in addition to contacts.

M. I. Karlinskaia and M. N. Siniagina (Academy of Communal Economy) in a paper "Application of no noncontact magnetic elements in relay remote control devices" described the results of the work conducted since 1954 in the Academy of Communal Economy on the application of feedback choke magnetic amplifiers in low-traffic RC-CO equipment.

V. M. Ostianu (Institute of Automation and Remote Control of the Academy of Sciences, USSR) showed in her paper "Synthesis of circuits with multipositional elements" that the operation of circuits with multipositional elements is represented by double-valued functions of many-valued variables. For the synthesis, analysis and transformation of such circuits, therefore, the known methods of the theory of relay circuits can be applied.

B. M. Rakov (VINITI AN SSSR) read a paper entitled "A method of constructing relay operating circuits which contain contacts and resistances" in which he proposed a design method of calculating and constructing relay circuits which permits one in certain cases to reduce the number of contacts by employing resistances.

Several papers were devoted to methods of synthesizing various types of circuits.

Engineer G. Ioanin (IPROMET Bucharest) dealt in his paper "Operation of relay circuits with real contacts" with reasons leading to misalignments in some of the circuits when ideal contacts in structural formulas were replaced by real ones. The proposed method provides the facility of altering the circuit in such a way as to make it work with actual relays.

V. I. Ivanov (Institute of Automation and Remote Control of the Academy of Sciences, USSR) in his paper entitled "Synthesis of cyclic relay-contact circuits with a single-type structure," outlined four basic operation sequences of the final control elements of this type of circuits and proved two theorems leading to analytical relationships which disclose the regular nature of their operation.

A. D. Kharkevich (Laboratory of the Academy of Sciences, USSR for solving scientific problems of wire communications) read a paper entitled "Switching circuits." Switching circuits represent one of the forms of sequential (m, n)-poles intended for use with objects of the same type, belonging to one or several groups. The author characterized various methods of constructing switching circuits and pointed out the basic problems in the theory of switching circuits.

In his second paper entitled "Application of the method of probability graphs for analyzing switching circuits" A. D. Kharkevich discussed an approximate method of determining some characteristics of switching circuits. By means of this method the number of trunks for group selectors in a telephone substation and optimum coefficients for extending a stage of tandem selectors of a crossbar system were found.

In a paper by a number of collaborators of the Laboratory of the Academy of Sciences, USSR for solving scientific problems of wire communications, A. A. Arkhangelskii, V. F. D'iachenko and V. G. Lazareva entitled "Application of the relay-contact circuit theory for analyzing and synthesizing telephone circuits" the experience of the LPC of the Academy of Sciences, USSR in applying the relay circuit theory in analyzing the operation of the existing and designing new ACO's was shown in a number of practical examples.

D. I. Shnarevich (Leningrad Institute of Railway Transport) in his paper "Transformation of relay circuits with additional windings" discusses equivalent transformations of relay circuits with additional windings.

In view of the fact that the analysis and synthesis of complex circuits demands a large expenditure of time and labor, the design of equipment for making this process automatic is of immediate interest. Four papers dealt with this question.

Corresponding Member of the Academy of the Hungarian PR L. Kal'mar (Mathematical Institute of the Szeged University) examined in his paper "The Szeged logical computer and some of its applications" described the principles of operation and constructional peculiarities of the electro-mechanical computer for solving logical problems built at the Mathematical Institute of the Szeged University. For a logical formula compiled by means of conjunctive, disjunctive and negative operations the computer determines a system of values for independent variables for which the formula assumes one of two values.

The author also noted the practical applications of the computer, by means of which it becomes possible to analyze three-pole contact circuits, and to help in synthesizing telephone circuits, SB Central Offices, etc.

P. P. Parkhomenko (Institute of Automation and Remote Control of the Academy of Sciences of the USSR) reviewed in his paper "Automation of relay circuit analytical processes" various methods of analyzing single and multicycle circuits. The author showed that the method of splitting the circuit into constituent units is valid for analyzing relay-contact circuits of any complexity or configuration and in this sense is the most generally applicable. The possibility of relatively simple application of the principle of splitting a circuit into its constituent units,and above-mentioned advantages,make this method preferable to others.

The new theoretical circuit solutions found by the author eliminated the disadvantages which were inherent in the previously made devices for mechanizing relay circuit analysis. As the result the author was able to make a computer consisting of 20 elements. The computer is capable of analyzing both single and multicycle circuits, moreover for multicycle circuits it is possible directly to obtain switching tables. Relay circuits with noncontact elements can also be investigated on the analyzer by means of introducing their relay-contact equivalents.

The analyzed circuit is connected to the analyzer by assembling it with cord pairs on a special switchboard multiple. The results of the analysis are either displayed on a lighted board (with manual control) or printed automatically on a punched card or on paper. The computer can compare two circuits by their conditions of functioning or not functioning, it can split them along sections and solve logical problems.

T. T. Tsukanov (Tomsk Institute of Railway Transport Engineers) described in his paper "Matrix analyzer of relay-contact circuits" the principles of operation and constructional peculiarities of his computer for analyzing relay-contact circuits.

The analyzed circuit is fed into the analyzer in a matrix form, which facilitates the connecting and transformation processes and greatly simplifies the construction of the analyzer. The task of the computer consists in an automatic search for basic circuits and their reproduction in the form of Boolian products for a particular characteristic.

The matrix analyzer can be used with single and multicycle circuits. The matrix analyzer can solve other types of problems as well, providing the problem can be reduced to determining basic routes in a geometric construction (or compiling particular characteristics). In particular the analyzer can be used for multiplying polynomials and matrices in a letter form.

In the paper by collaborators of the Laboratory of the Academy of Sciences, USSR, for solving scientific problems in wire communications A. A. Arkhangel'skii, V. G. Lazarev and V. N. Roginskii entitled "Mechanization of the relay circuit synthesizing process" the authors describe the principles of operation of an automatic computer for synthesizing contact (1, k)-poles. The computer is based on a graphic method of contact circuit representation. The synthesized circuit of a contact (1, k)-pole is set on $\underline{k}$ assemblies of conditions under which each circuit must be closed (obligatory numbers) or may be closed (conditional numbers). The problem is set on the computer by appropriate turning of three-position switches. From all the possible answers the computer chooses the optimum one from the point of view of the number of contacts.

Following an exhaustive discussion the conference adopted a decision noting that the present stage of the development of automatic and remote control devices is characterized by an increasing application of complex devices performing many functions and based on the principle of relay operation.

Further development of a scientific theoretical foundation in the sphere of relay devices is of the utmost importance.

The theory of relay devices is at present being developed in a number of countries and has already successfully solved a number of theoretical problems, and has established practical methods of synthesis, analysis and equivalent transformation of relay device structures.

At present this theory should be regarded as the general theory for controlling and monitoring devices with discrete operation, characterized by the transmission and transformation of effects with two or more fixed values.

The practical application of the theory of relay devices has shown its great efficiency. The solution of practical problems in relay technology by means of this theory leads to a more rational construction.

The conference suggested the basic lines of development for the theory of relay devices. The decision notes among other things the necessity of a simultaneous development of the theory of signals and the structural theory in order to obtain the most rational construction.

A very important problem in the theory of relay circuits is its extension to cover the structure of devices with various noncontact elements.

The conference noted the first successes in the development of automatic computers for synthesizing and analyzing relay devices.

The conference pointed to the necessity of substantially developing the work in the sphere of the theory of structures both by utilizing the available resources and by obtaining new collaborations in the first place from union republic academies of sciences, schools of higher learning and universities.

With the object of achieving international coordination in the work on the theory of relay acting devices and with the aim of obtaining a wider discussion of the possibilities and achievements in this sphere the conference found it necessary to establish an international federation for dealing with the theory of relay devices.

A number of measures were suggested with respect to training of personnel, issuing information and in particular printing in the shortest possible time the conference transactions.

An exhibition of Soviet and foreign literature on the theory of relay devices and its practical application was organized at the conference. Photographic reproductions of the first articles and books on the subject which have now become a bibliographical rarity were also shown at the exhibition.

Modern relay device models were on show in the conference hall.

The conference helped to establish contact between the mathematicians and engineers and strengthen the ties between the Soviet and foreign scientists working on the problems of the theory of relay devices.

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